Chapter 6
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in Hydrologic Models

CHAPTER 6: SIMULATION OF RUNOFF GENERATION IN HYDROLOGIC MODELS

The sections above have focused on understanding the rainfall-runoff processes. This understanding forms the basis for a number of numerical models that represent runoff process in a conceptual way and are used to continuously simulate runoff generation in a research or operational setting. The essential feature of a simulation model is that it produces an output or series of outputs in response to an input or series of inputs. In the case of a rainfall-runoff model the inputs are characteristics of the watershed being modeled, such as drainage area and channel network geometry (size and length), topography, soil and land use characteristics and a time series of surface water input. The output is a time series of streamflow at an outlet location. Lumped models treat a whole catchment, or a significant portion of it, as a single unit, with inputs, internal state variables and outputs representing the hydrologic processes over the catchment as a whole. Distributed models divide the catchment into a number of sub areas; simulate each of them, and the interactions between them separately, maintaining different state variables for each model element, then combine the outputs to obtain catchment response. The distinction between lumped and distributed may amount to one of scale. A lumped model can be applied over a set of small watersheds that comprise a catchment to obtain a distributed model. However there is also often a difference in representation of the processes between lumped and distributed models, with distributed models being based more on the basic physical equations used to describe the processes involved and taking advantage of physically measurable attributes of the watershed, whereas lumped models use a more conceptual representation of the rainfall runoff process. There is a vast literature on hydrologic modeling that we do not address here (see Freeze and Harlan, 1969; Beven, 1989; Beven and Binley, 1992; Grayson et al., 1992a; Grayson et al., 1992b; Beven, 2000 and the descriptions in a number of hydrology texts), but we do review two of the more prominent rainfall runoff models.

TOPMODEL is a rainfall-runoff model (Beven and Kirkby, 1979) that takes advantage of topographic information (specific catchment area and wetness index) related to runoff generation, although Beven et al. (1995) prefer to consider TOPMODEL as not a hydrological modeling package, but rather a set of conceptual tools that can be used to reproduce the hydrological behavior (in particular the dynamics of surface or subsurface contributing areas) of catchments

in a distributed or semi-distributed way. The National Weather Service River Forecast System (NWSRFS) is another such model that is more conceptual in nature involving the accounting for water storage in a number of conceptual stores representing the hydrologic state of components of the watershed involved in rainfall-runoff processes. In this section we review some of the key ideas in these models as they are related to rainfall runoff processes so that the users of these models can appreciate the physical basis for conceptual process representations that these models use.

TOPMODEL

TOPMODEL is fully described by Beven et al. (1995). Streamflow is separated into surface runoff generated by surface water input on saturated contributing areas and subsurface downhill flow comprising baseflow and return flow. TOPMODEL uses four basic assumptions to relate down slope flow from a point to discharge at the catchment outlet.

A1. The dynamics of the saturated zone are approximated by successive steady state representations.

A2. The recharge rate r [m/hr] (equation 2) entering the water table is spatially homogeneous.

A3. The effective hydraulic gradient of the saturated zone is approximated by the local topographic surface gradient S ($\tan\beta$ is the notation most common in TOPMODEL descriptions).

A4. The effective down slope transmissivity T of a soil profile at a point is a function of the soil moisture deficit at that point. This is commonly based on an exponential decrease of hydraulic conductivity with depth, but Ambroise et al. (1996) generalized this to also include linear and parabolic relationships.

Assuming steady state with spatially homogeneous recharge rate (A1 and A2), the down slope subsurface flow rate per unit contour width is given as (equation 2, Figure 49)

$$q = r\,a \tag{75}$$
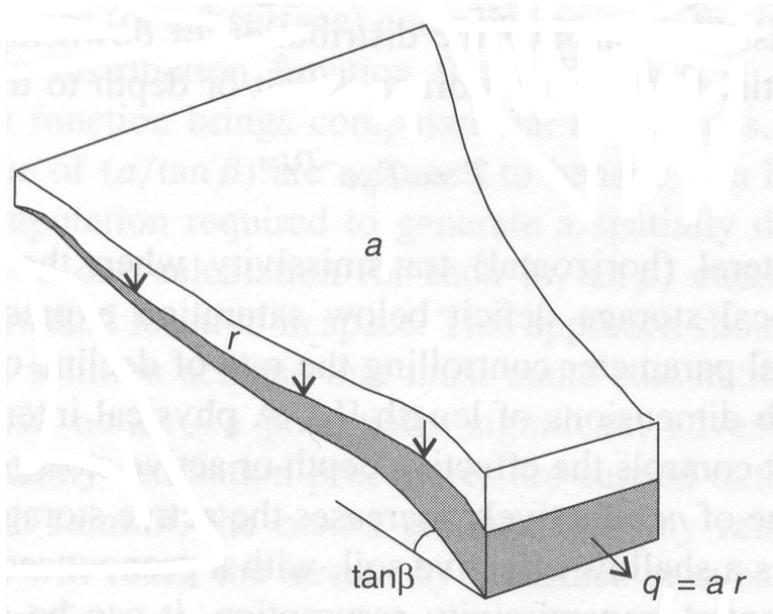
Figure 49. Definition of the upslope area draining through a point within a catchment (from Beven, 2000).

Assuming an exponential decrease of hydraulic conductivity with depth, z,

$$K(z) = K_o e^{-fz} \tag{76}$$

and soil moisture deficit quantified in terms of the depth to the water table z_w the down slope transmissivity of the saturated part of the soil profile below the water table is

$$T(z_w) = \int_{z_w}^{\infty} K(z)dz = \int_{z_w}^{\infty} K_0 e^{-fz}dz = \frac{K_0}{f} e^{-fz_w} = T_0 e^{-fz_w} \tag{77}$$

Here K_o is the hydraulic conductivity at the surface and f sensitivity a parameter that quantifies how rapidly hydraulic conductivity decreases with depth. $T_o = K_o/f$ is the transmissivity of the soil profile. If an effective porosity θ_e is assumed, the soil moisture deficit D [m] can be approximated (equation 38) as

$$D = \theta_e z_w \tag{78}$$

This assumes moisture content at the residual moisture content above the water table. Equation (77) can then be written in terms of soil moisture deficit rather than depth to the water table as

$$T(D) = T_0 e^{-fz_w} = T_0 e^{-fD/\theta_e} = T_0 e^{-D/m} \qquad (79)$$

where $m = \theta_e/f$. Some descriptions of TOPMODEL take equation (79) as the starting point for assumption A4, with m a parameter controlling the rate of decline of transmissivity with increasing soil moisture deficit, while other TOPMODEL descriptions start from assuming (76).

Now assuming (A3) that the hydraulic gradient is equal to the local surface slope

$$q = T(D) \, S = T_0 e^{-D/m} \, S \qquad (80)$$

By combining (75) and (80) it is possible to calculate the local soil moisture deficit D

$$r \, a = T(D) \, S = T_0 e^{-D/m} \, S \qquad (81)$$

therefore

$$D = -m \ln\left(\frac{r \, a}{T_0 S}\right) \qquad (82)$$

This is the soil moisture deficit that results from the depth to the water table naturally adjusting so that transmissivity of the soil profile below the water table is equal to the lateral drainage of steady state recharge from the hillslope above. Under steady state drainage, r is also the per unit area baseflow discharge from the watershed, so equation (82) provides the capability to calculate soil moisture deficit at each point in a watershed based on the overall moisture condition of the watershed as expressed by baseflow r and the topographic and soils parameters a, T_o and S. TOPMODEL uses a series of steady state representations (A1) to represent the dynamics of soil moisture deficit D over a watershed using equation (82). Equation (82) may be written

$$D = -m\ln(r) - m\ln\left(\frac{a}{T_oS}\right)$$

$$= -m\ln(r) + m\ln(T_o) - m\ln\left(\frac{a}{S}\right) \tag{83}$$

$$= -m\ln(r) - m\gamma = -m\ln(r) + m\ln(T_o) - m\lambda$$

where we have denoted

$$\lambda = \ln\left(\frac{a}{S}\right) \quad \text{and} \quad \gamma = \ln\left(\frac{a}{T_oS}\right) \tag{84}$$

λ is a topographic wetness index that quantifies the dependence of soil moisture deficit on topographic parameters a and S. γ is a soil and topographic wetness index that quantifies the dependence of soil moisture deficit on soil and topographic parameters T_o, a and S. λ and γ are sometimes referred to as similarity indices because of the assumption that all locations with the same λ or γ have equivalent hydrologic behavior, in terms of soil moisture deficit and generation of runoff by saturation excess. A further consequence of this similarity is that the specific spatial locations of points with a specific λ or γ do not matter. The probability distribution as represented for example using a histogram is sufficient to describe the hydrologic response of a watershed using TOPMODEL.

The mean catchment soil moisture deficit $\overline{D}$ is obtained by spatially averaging (82) over the entire area A of the catchment.

$$\overline{D} = \frac{1}{A}\int -m\ln\left(\frac{ra}{T_oS}\right)dA = -m\ln(r) - m\overline{\gamma}$$

$$= -m\ln(r) + m\overline{\ln(T_o)} - m\overline{\lambda} \tag{85}$$

where the over bar is used to denote spatial averaging. Subtracting equation (85) from (83) one obtains

$$D = \overline{D} - m(\gamma - \overline{\gamma}) \tag{86}$$

for the case where T_o is spatially variable, and

$$D = \overline{D} - m(\lambda - \overline{\lambda}) = \overline{D} - m\left(\ln(a/S) - \overline{\lambda}\right) \qquad (87)$$

for the case where T_o is assumed to be spatially constant. This explicitly establishes the basis for the topographic wetness index $\ln(a/S)$.

Equations (86) and (87) provide a way to calculate the soil moisture deficit D at each point in a catchment given the average soil moisture deficit, a soil parameter m, and the difference between the average topographic index and the local topographic index (λ or γ). m can be estimated from soil characteristics or calibrated based on the catchment recession curve. λ or γ can be estimated from a topographic map (and a map of soil characteristics in the case of γ), and we can keep track of $\overline{D}$ by water balance accounting. If D is less than 0, the soil is completely saturated and any rain on the surface will become overland flow by the process of saturation excess runoff generation. TOPMODEL therefore, through the topographic index, provides an explicit modeling of the saturation excess runoff process. This is the key distinguishing feature of TOPMODEL.

In equations (75) and (81) the rate of recharge to the water table, r, was assumed to be spatially homogeneous. With the steady state assumption, integrating this over the watershed, the subsurface response per unit area is also r. This can be expressed in terms of the average deficit by replacing D in equation (81) with (86) or (87) to obtain

$$r = e^{-\overline{D}/m} e^{-\overline{\gamma}} = T_o e^{-\overline{D}/m} e^{-\overline{\lambda}} \qquad (88)$$

This is the mean subsurface discharge from the watershed. The last expression above is for the case when T_o is spatially constant. Thus in TOPMODEL the subsurface flow is controlled by the soil characteristics T_o and m, topography (λ or γ) and the average saturation deficit of the catchment. Equation (88) gives the drainage rate of basin average soil moisture as an exponential function of the soil moisture deficit. During periods where there is no precipitation integration of this equation describes the recession curve. This property provides a way to estimate the parameter m from analysis of baseflow recession curves. This is not all of TOPMODEL, but is a summary of the main conceptual points. Added to the components above are other standard components of the water budget: interception, infiltration excess runoff generation, evapotranspiration,

snowmelt and channel routing. Several computer model implementations of TOPMODEL exist that incorporate the runoff generation components described above with representation of these other processes to provide a complete continuous simulation modeling capability.

Example 4. This example focuses only on the runoff generation aspects of TOPMODEL that can be fairly easily calculated using a Geographic Information System (GIS). Consider the Spawn Creek watershed shown in Figure 50, with parameters K_o=10 m/hr, f=5 m^{-1}, θ_e=0.2.
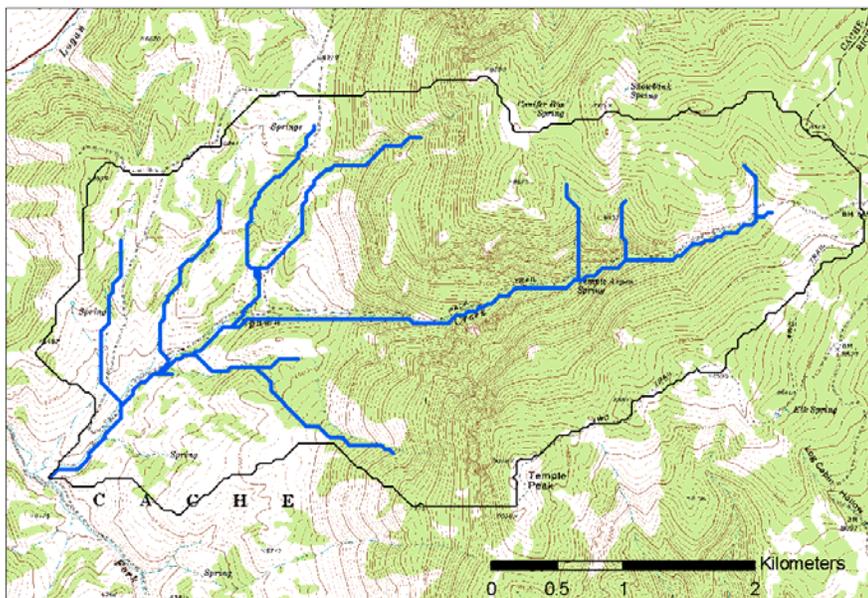
Figure 50. Channel network and drainage area delineated from a digital elevation model of the Spawn Creek area in Logan Canyon UT.

Here we assume that K_o is spatially uniform. Assume that the baseflow prior to a rainfall is Q_b = 0.8 m^3/s. We want to calculate the saturated area using TOPMODEL, and the quantity of runoff from 25 mm of rainfall. As part of calculating this we will also calculate the expansion of the saturated area during the rainfall and the map these areas. GIS methods are beyond the scope of this module, but the relatively simple capability used here can be readily applied using ArcGIS and the TauDEM software (Tarboton, 2002). Figure 50 shows the channel network and drainage area delineated from a digital elevation model of the Spawn Creek area in Logan Canyon UT. Although this is real terrain the rest of the parameters

used in this example are synthetic, for illustration purposes, and not related to physical conditions in this watershed.

This example watershed was delineated using TauDEM software with a 30 m grid digital elevation model from the National Elevation Dataset. The watershed as delineated comprises 15890 grid cells. The drainage area is therefore A=15893 x 30 x 30 = 14,303,700 m^2 = 14.3 km^2. The per unit area baseflow is therefore r=Q_b/A = 0.8/14,301,000 = 5.59 x 10^{-8} m/s. Multiplying by the number of seconds in an hour r = 5.59 x 10^{-8} m/s x 3600 s/hr = 0.0002 m/hr = 0.2 mm/hr. GIS methods were used to calculate the slope, specific catchment area and wetness index ln(a/S) for each grid cell in this area as illustrated in Figure 51.

Note that there are a few gaps in the grid where wetness index has been calculated. These are locations where the slope S is 0 and ln(a/S) is therefore mathematically undefined. These locations need to be treated separately. In TOPMODEL theory the depth to the water table is based on the adjustment of lateral transmissivity to balance the drainage from upslope under a topographic drainage. Where there is no topographic gradient (slope is 0) there can be no lateral soil moisture drainage in the theory, so these grid cells are always saturated. There are 81 grid cells with 0 slopes. Averaging ln(a/S) over the non zero slope grid cells we obtain $\overline{\lambda}$=6.9. The transmissivity parameter is $T_o = K_o/f = 10/5 = 2$ m^2/hr and m = $\theta_e/f = 0.2/5 = 0.04$ m^{-1}. The average soil moisture deficit is evaluated using equation (85)

$$\overline{D} = -m\ln(r) + m\,\overline{\ln(T_o)} - m\,\overline{\lambda}$$

$$= \text{-0.04 } \ln(0.0002) + 0.04 \ln(2) - 0.04 \text{ x } 6.9 = 0.092 \text{ m}$$

The soil moisture deficit at each location may then be evaluated using equation (87).

$$D = \overline{D} - m\left(\ln(a/S) - \overline{\lambda}\right) = 0.092 \text{ - 0.04 x } (\ln(a/S) \text{ - } 6.904)$$

Equivalently one could have computed D directly using equation (82) without the intermediate steps of evaluating $\overline{\lambda}$ and $\overline{D}$. Figure 52 shows a map of D.

Figure 51. Slope, Specific Catchment Area and Wetness Index for Spawn Creek.
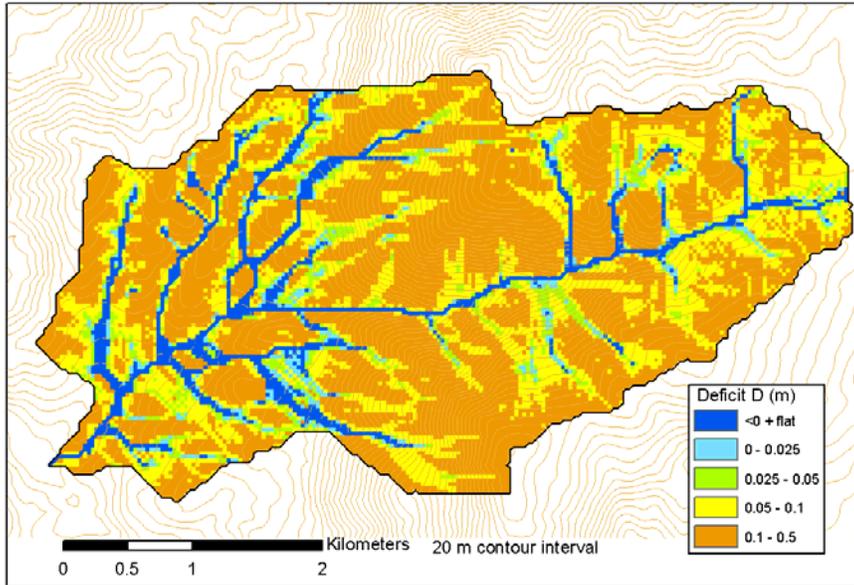
Figure 52. Soil Moisture Deficit for Spawn Creek calculated using TOPMODEL.

At the average soil moisture deficit calculated above, the dark blue shaded areas in figure 52 are already saturated and will generate saturation excess from any surface water input. There are 1246 grid cells with $D \leq 0$ which when combined with the 81 flat grid cells gives a saturated area of $(1246+81)$ x 30 x 30 = 1,194,300 m^2 = 1.2 km^2 or 1.2/14.3 = 8.3% of the watershed. The lighter blue areas have soil moisture deficit less than 0.025 m = 25 mm so will become saturated during 25 mm of surface water input. There are 546 grid cells in the range $0 < D \leq 0.025$ m that have an area 546 x 30 x 30 = 491,400 m^2 or 3.4% of the watershed. The green, yellow and brown areas require surface water inputs greater than 25 mm to generate runoff by saturation excess, but may generate runoff by infiltration excess depending upon the surface water input rate and infiltration capacity and could be calculated using the Horton, Philip or Green-Ampt model described above (examples 1-3) . Considering only the runoff due to saturation excess from the 25 mm of surface water input, 25 mm of runoff is generated over 8.3% of the watershed. Over 3.4% of the watershed something between 0 and 25 mm of runoff is generated. This is evaluated using the GIS raster calculator functionality evaluating 0.025 - D and averaging this for the grid cells for which $0 < D \leq 0.025$ m. The result is 0.0113 m = 11.3 mm. The total basin runoff volume is therefore:

$$0.025 \text{ x } 1,194,300 + 0.0113 \text{ x } 491,400 = 35,410 \text{ m}^3$$

The corresponding per unit area runoff depth is:

$35,410/14,303,700 = 0.0025$ m $= 2.5$ mm.

This represents a runoff ratio of $2.5/25 = 0.1$ or 10%.

National Weather Service River Forecast System

The Sacramento Soil Moisture Accounting model has become a major tool in the National Weather Service River Forecast System (NWSRFS). The model is based on the generalized hydrologic model (Burnash et al., 1973) depicted in Figure 53 intended to represent significant hydrologic processes in the headwaters of a watershed in a conceptual manner. It is beyond the scope here to describe the model completely, but we present a brief overview of the key ideas involved in the representation of rainfall – runoff processes.
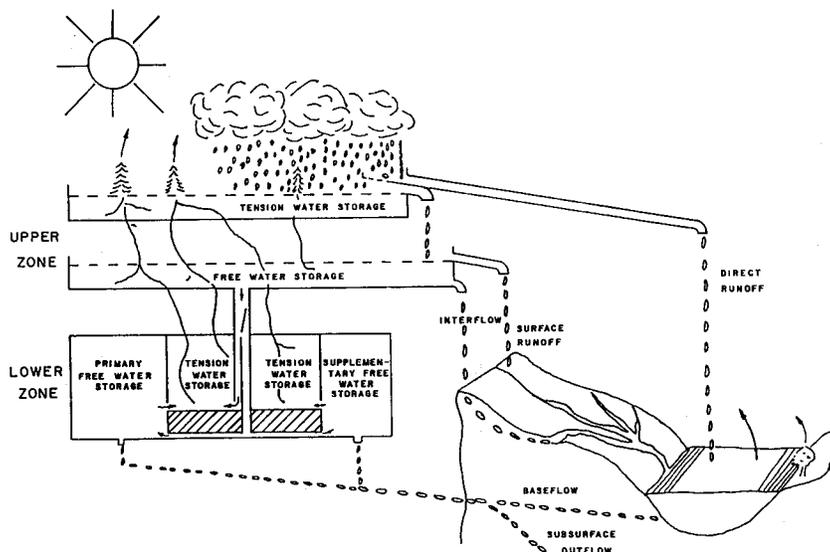
Figure 53. Generalized Hydrologic Model that serves as the basis for the National Weather Service River Forecast System (from Burnash et al., 1973).

In the Sacramento model incoming rainfall is distributed between interception, impervious areas such as lakes and streams and water input to the upper zone. The upper zone consists of interception storage, tension water storage, representing water held by capillary forces and free water storage that supplies percolation to the lower zone as well as interflow and surface water runoff. Model

parameters, which need to be user supplied or calibrated, govern the rate of water movement between the zones and the capacity of the storage zones. Surface runoff occurs when the upper zone capacity is completely filled, due to precipitation rate being in excess of the percolation rate for sufficient time.

To represent the spatial variability of infiltration capacity a linear cumulative distribution function shown as a line from the origin to point b in Figure 54 is used. The position of this line is varied, by varying the value of b, as a function of the ratio of moisture in the lower-zone storage (LZS) to the nominal capacity of this zone (LZSN). The net infiltration to the lower zone is the hatched trapezoid of Figure 54, and is a function of the current soil moisture ratio (LZS/LZSN) and the supply rate X. Interflow is calculated by a similar process. Line B of Figure 54 divides the rainfall excess triangle into two portions, surface runoff and interflow. The position of line B is fixed by multiplying b by a factor c which is greater than 1 and is also a function of LZS/LZSN. The fraction of interflow increases as soil moisture increases.
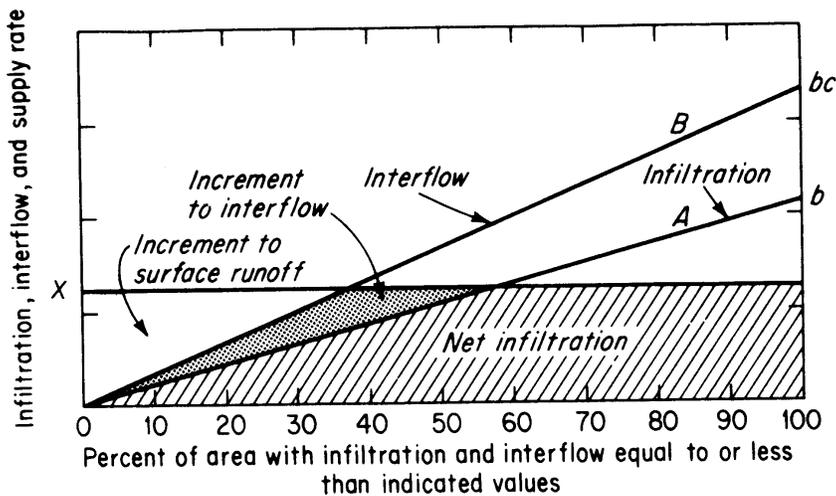
Figure 54. Infiltration-interflow function used in the Sacramento Model (from Linsley et al., 1982).

A complete flow diagram depicting all the processes represented is shown in Figure 55. There are a large number of parameters representing the storage capacities of the various stores and characterizing the rates of flow between these stores. These parameters need to be estimated for each specific watershed through calibration that takes measured precipitation inputs and minimizes a

measure of the difference between modeled and observed streamflow. Considerable research on parameter optimization exists and the reader is referred to recent papers in this area for a perspective on the methods available (e.g. Beven and Binley, 1992; Kuczera, 1997; Gupta et al., 1998; Beven, 2000).
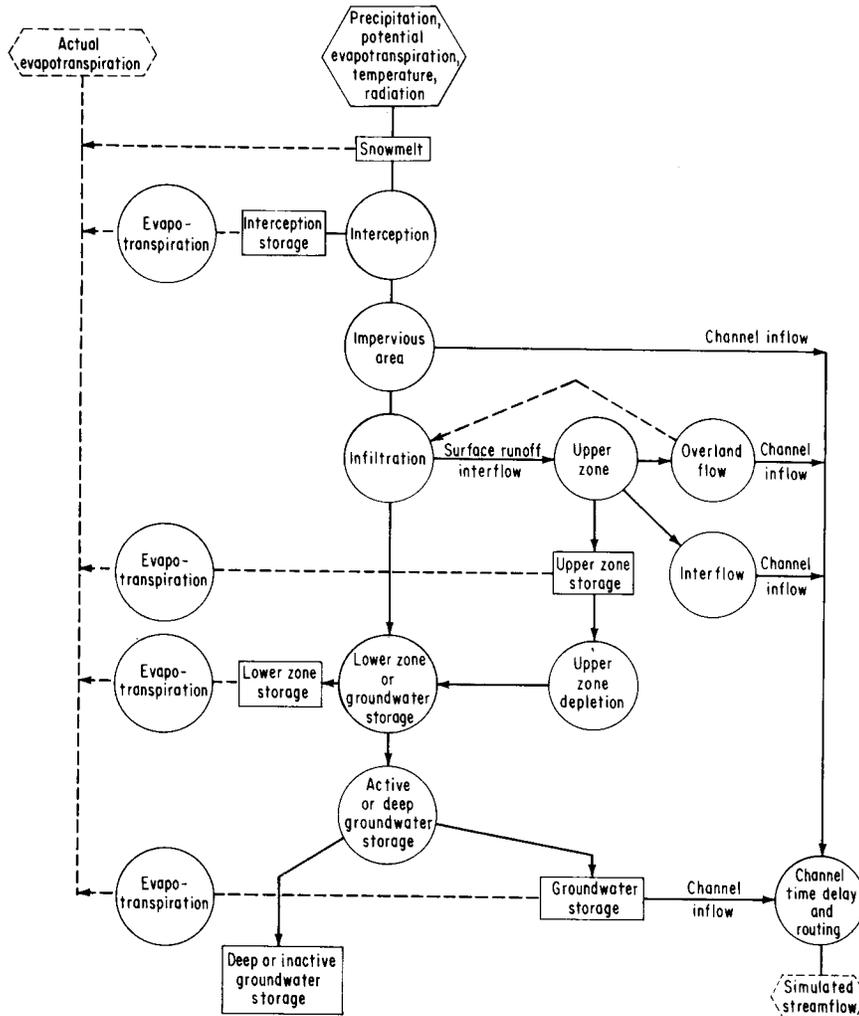
Figure 55. Flow diagram of the Sacramento Model (from Linsley et al., 1982).

Exercises

1. The probability distribution of wetness index represented by a histogram is sufficient to describe the hydrologic response of a watershed using TOPMODEL. Consider a watershed that has TOPMODEL wetness index ln(a/S) distributed according to the histogram shown
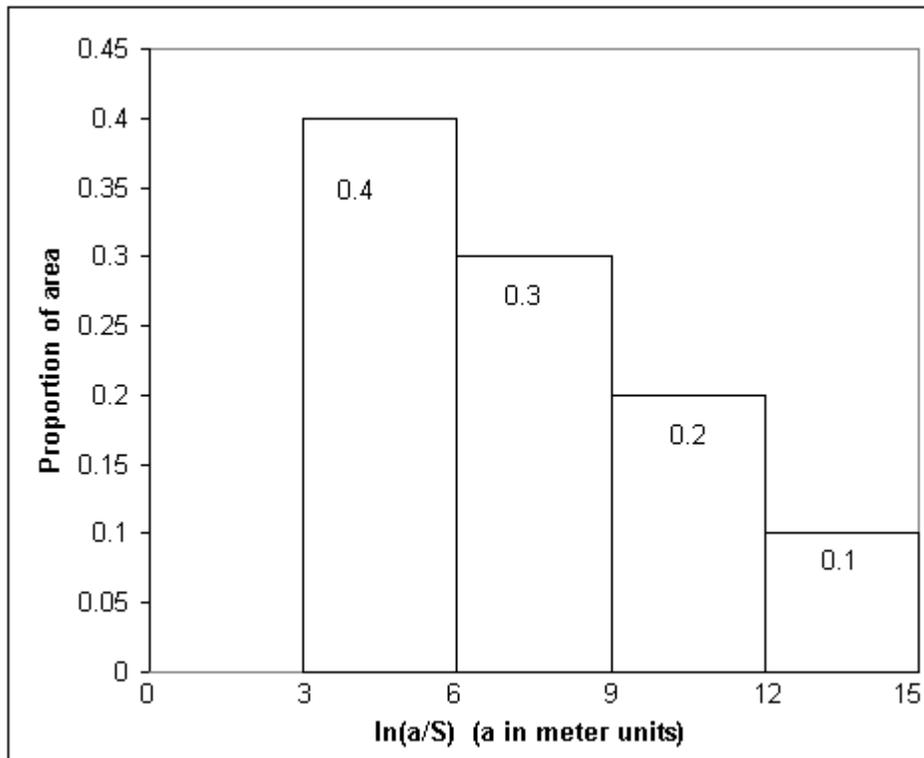
The watershed has TOPMODEL parameters: Spatially homogeneous K_o= 4 m/hr, f= 2 m^{-1}, antecedent baseflow Q_b = 12 m^3/s, drainage area = 400 km^2, effective porosity of unsaturated zone = 0.2.

a) Estimate the TOPMODEL $\overline{\lambda}$ parameter for this watershed by averaging the histogram above.
b) Estimate the recharge parameter r by dividing baseflow by the drainage area and expressing the result in m/hr.
c) In TOPMODEL locations with wetness index ln(a/S) greater than a threshold are saturated to the surface. This threshold can be obtained from setting D=0 in equation (82) or (83) and solving for ln(a/S). Report the threshold ln(a/S) above which saturation occurs for the antecedent conditions given.

d) The fraction of watershed that is initially saturated is obtained by integrating the histogram over values larger than the threshold determined in (c). Report the fraction of watershed that is initially saturated.

e) Following infiltration of rainfall the saturated area expands to encompass all locations where D was less than the rainfall. Use equation (82) or (83) to determine the threshold ln (a/S) corresponding to D=0.025 m (25mm).

f) Integrate the histogram over values larger than the threshold determined in (e) to determine the fraction of watershed that is saturated at the end of a 25 mm rainstorm.

g) Estimate the volume of runoff from 25 mm of rain, by summing the runoff from the initially saturated area and the area that becomes saturated during the storm. Report your answer on a per unit total area basis.

h) Equation (88) gives the baseflow in terms of average soil moisture deficit $\overline{D}$. This decreases by 25 mm due to infiltration during the storm and this equation may be used to calculate the corresponding increase in r which corresponds to an increase in baseflow. Use equation (88) to determine the baseflow that you expect after the direct runoff hydrograph from the 25 mm of rainfall has receded. Report your result in m^3/s.

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