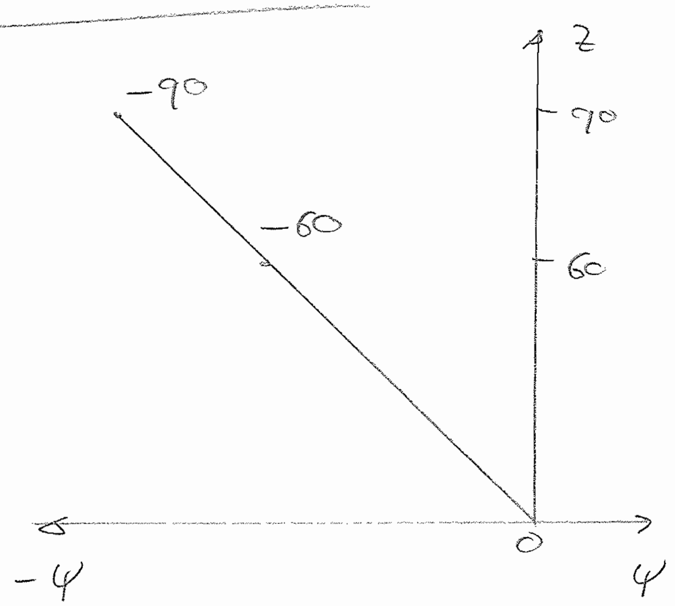
# 2011 FINAL SOLUTION

1. a)



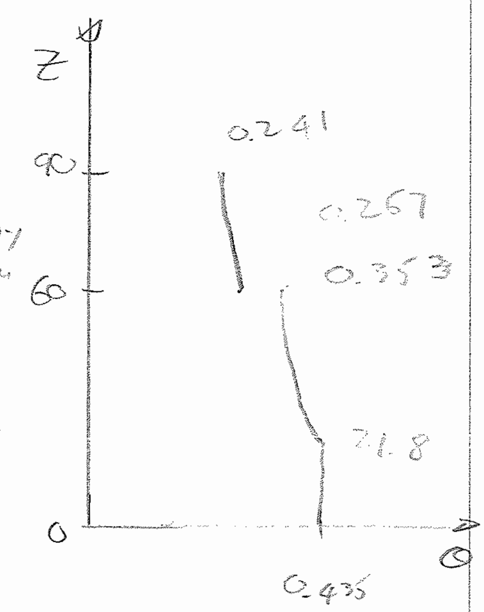

b) IN SANDY LOAM $|\psi_0| = 21.8$ cm

∴ TOP OF CAPILLARY FRINGE AT 21.8 cm ⟹ ABOVE WATER TABLE

c) $$|\psi| = |\psi_0|\left(\frac{\theta}{n}\right)^{-b}$$

$$\therefore \theta = n\left(\frac{|\psi|}{|\psi_0|}\right)^{-1/b}$$

| z | n | $\lvert\psi\rvert$ | $\lvert\psi_0\rvert$ | θ | |
|---|---|---|---|---|---|
| 0 | 0.435 | | 21.8 | 0.435 | |
| 21.8 | | | 21.8 | 0.435 | SANDY LOAM |
| 60 | 0.435 | 60 | 21.8 | 0.353 | |
| 60 | 0.395 | 60 | 12.1 | 0.267 | SAND |
| 90 | 0.395 | 90 | 12.1 | 0.241 | |



d) $SMD = \int_{z_1}^{z_2} (n - \theta(z))\, dz$

$= \int_{z_1}^{z_2} n\left(1 - \left(\frac{z}{|\psi_a|}\right)\right)^{-1/b} dz$

$= n(z_2 - z_1) - n I$

where $I = \int_{z_1}^{z_2} \left(\frac{z}{|\psi_a|}\right)^{-1/b} dz$

$= \frac{1}{|\psi_a|^{-1/b}} \frac{z_2^{1-1/b} - z_1^{1-1/b}}{1 - 1/b}$

For bottom layer

$|\psi_a| = 21.8$ $b = 4.9$ $n = 0.435$

$z_2 = 60$ $z_1 = 21.8$ $1 - \frac{1}{b} = 0.795$

$\therefore$ $I = 33.92$

SMD = 1.86 cm

For top layer

$|\psi_a| = 12.1$ $b = 4.05$ $n = 0.395$

$z_2 = 90$ $z_1 = 60$ $1 - \frac{1}{b} = 0.753$

$I = 19.16$

SMD = 4.28 cm

Combined SMD = 6.14 cm

e) Time to ponding.

Rain rate = $8/24 = \frac{1}{3}$ cm/h

This is less than $K_{sat}$ for both layers so no infiltration excess

Ponding will occur when the soil is filled.

$$t_p = \frac{6.14 \text{ cm}}{1/3 \text{ cm/hr}} = \underline{18.4 \text{ hr}}$$

f) Runoff generated = 8 - 6.14

= $\underline{1.86 \text{ cm}}$

2 a) $e_s = 0.611 \exp \frac{17.3 T}{237.3 + T}$ $T = 18°C$

$= 2.069$ kPa

b) $\rho_v = \rho_a \frac{0.622 e}{p}$ Diagram D-9

$\rho_{vsat} = 1.13 \times \frac{0.622 \times 2.069}{95}$

$= 0.0153$ $kg\ m^{-3}$

$W_a = 0.54$

$\therefore \rho_v = 0.54 \times \rho_{vsat}$

$\Delta\rho_v = (1 - W_a)\rho_{vsat} = 0.46 \times 0.0153$

$= 0.0070$ $kg\ m^{-3}$

c) $f(\Delta\rho_v) = 1 - 66.6 \times 0.0070$

$= 0.531$ This is consistent with 0.528 in the spreadsheet.

d) $C_{leaf} = C^{*}_{leaf} \times f_k f_\rho f_T f_\theta$

$= 5.3 \times 0.399$

$= 2.11$ mm/s

e)

$$ET = \frac{\Delta(K+L) + \rho_a c_a C_{at} e_a^*(1-w_a)}{\rho_w \lambda_v \left[\Delta + \gamma\left(1 + \frac{C_{at}}{C_{can}}\right)\right]}$$

For consistency use $C_{at}$ in m day$^{-1}$

$$\frac{\cancel{kPa}\,\cancel{K^{-1}}\ \cancel{MJ}\ m^{-2}\ day^{-1} + \cancel{kg}\ m^{-3}\ \cancel{MJ}\ \cancel{kg^{-1}}\ \cancel{K^{-1}}\ m\ day^{-1}\ \cancel{kPa}}{\cancel{kg}\ m^{-3}\ \cancel{MJ}\ \cancel{kg^{-1}}\ \cancel{kPa}\ \cancel{K^{-1}}}$$

m day$^{-1}$

$$ET = \frac{0.13\times(16.5) + 1.13\times 1.005\times10^{-3} \times 2.69\times10^{-1}\times 86400 \times 2.069(1-0.54)}{1000\times 2.46\times\left[0.13 + 0.062\left(1 + \frac{269}{6.34}\right)\right]}$$

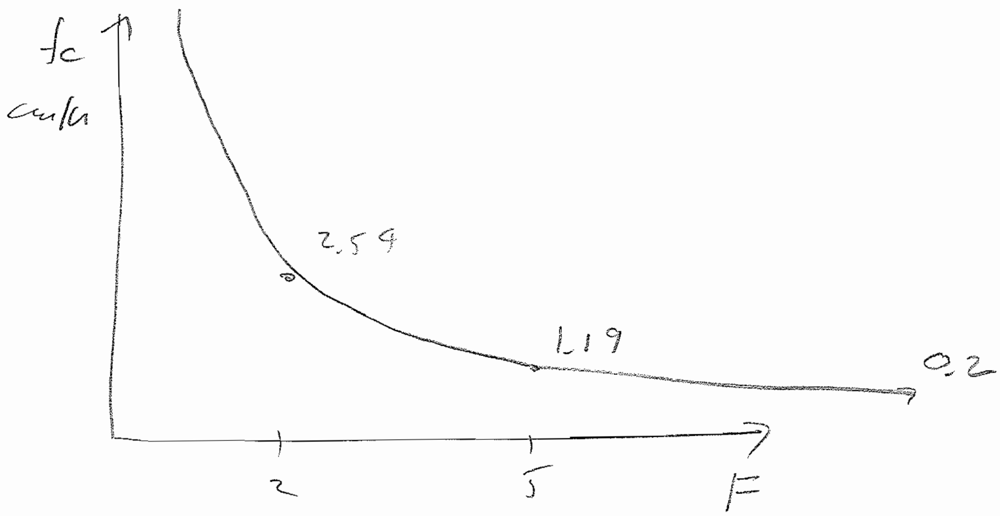
$= \underline{3.93\times10^{-3}\ \text{m day}^{-1}}$

$= \underline{3.93\ \text{mm/day}}$

3 a) $f_c(F) = K_p + \dfrac{K_p S_p}{\sqrt{S_p^2 + 4K_p F} - S_p}$

$K_p = 0.2$
$S_p = 3$

| $F$ | 0 | 2 | 5 | cm |
|---|---|---|---|---|
| $f_c$ | $\infty$ | 2.54 | 1.19 | cm/h |




b) when $\omega = 0.5$ cm/h

$F_p = \dfrac{S_p^2 (\omega - K_p/2)}{2(\omega - K_p)^2}$

$= 20$ cm

∴ No ponding in first cm [illegible]

when $\omega = 2$ cm/hr

$F_p = 2.639$ cm

1 cm infiltrated in first 2 hours in any
∴ 1.639 to infiltrate

$$\Delta t' = \frac{1.639}{2} = \underline{0.82\ h}$$

$$\therefore\ t_s = 2.82\ h \qquad F_s = 2.639$$

$$t_o = t_s - \frac{1}{4K_p^2}\left(\sqrt{S_p^2 + 4K_pF_s} - S_p\right)^2$$

$$= 2.125\ h$$

$$\therefore\ F = S_p(t-t_o)^{1/2} + K_p(t-t_o)$$

$$t = 4$$

$$F = \underline{4.48\ cm}$$

in first time step no runoff

in second time step $5 - 4.48 = \underline{0.518\ cm}$ of runoff.

ponding first at $\underline{t = 2.82\ h}$