Utah State University
Department of Civil and Environmental Engineering
CEE 3430 Engineering Hydrology

Test 2. Solution
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1. Consider a watershed with silty clay loam soil and Green-Ampt parameters given in Mays Table 7.4.1 (page 317). Consider a storm where 3 cm of rainfall occurs in 2 hours. Assume that the soil is initially dry with initial moisture content equal to residual moisture content. Calculate the following using the Green-Ampt approach.

    a) The infiltration capacity after 2 cm of infiltration
    b) The minimum infiltration capacity
    c) Time to ponding
    d) Depth of infiltration excess runoff generated from this storm

Solution
a) The parameters for silty clay loam from table 7.4.1 are
n=0.471, θ_e=0.432, ψ=27.3 cm, K = 0.1 cm/h

With initial moisture content equal to residual moisture content you can write immediately $\Delta\theta=\theta_e=0.432$. [Or work it out $\theta_r=n-\theta_e$ and $\Delta\theta=n-\theta_r=n-(n-\theta_e)=\theta_e$]

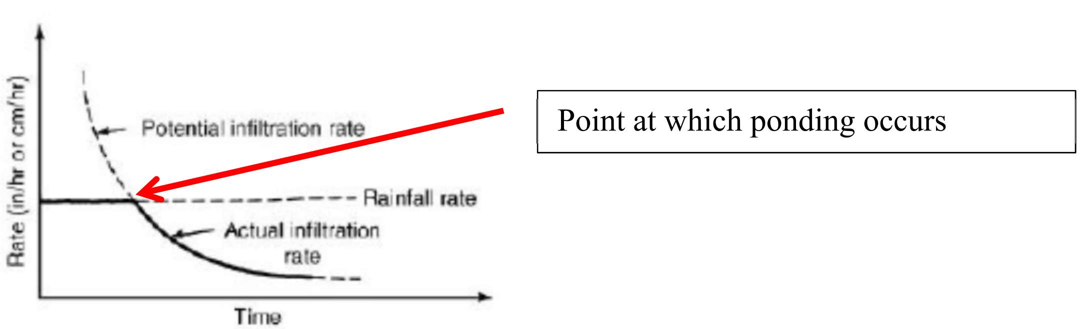
The fundamental state relationship in the Green-Ampt method is that infiltration capacity (potential infiltration rate) is a function of the depth of water infiltrated by equation 7.4.23. Using this here with infiltrated depth F=2 cm

$$f = K\left(1+\frac{\psi\Delta\theta}{F}\right) \quad \text{(eqn 7.4.23)}$$

$$f = 0.1\left(1+\frac{27.3\ \times 0.432}{2}\right) = \mathbf{0.6897\ cm/h}$$

b) The minimum infiltration capacity occurs as F increases towards infinity and is the saturated hydraulic conductivity, K = **0.1 cm/h**.

c) Equation (7.4.23) describes the decrease of infiltration capacity as the infiltrated depth increases. Ponding occurs when the infiltration capacity (also referred to as potential infiltration rate) decreases to the point where it equals the rainfall rate (see Mays fig 7.4.8)



at ponding

$$i = f = K\left(1 + \frac{\psi\Delta\theta}{F_p}\right)$$

and the depth of water that has infiltrated is $F_p = i\,t_p$
so

$$i = K\left(1 + \frac{\psi\Delta\theta}{i\,t_p}\right)$$

which can be solved to give

$$t_p = \frac{K\psi\Delta\theta}{i(i-K)}$$

This equation is given in example 7.4.4 so you could go straight to it. With the parameters given

$$i = \frac{3\ cm}{2\ h} = 1.5\ cm/h$$

$$t_p = \frac{0.1 \times 27.3 \times 0.432}{1.5(1.5-0.1)} = \mathbf{0.5616\ h}$$

d) To solve for infiltration after ponding use the more general form of equation 7.4.22 or 7.4.24 that accounts for the time shift along the time axis due to the time to ponding

$$F - F_p - \psi\Delta\theta \ln\left(\frac{F + \psi\Delta\theta}{F_p + \psi\Delta\theta}\right) = K(t - t_p)$$

This equation was given in class powerpoint slides. In some references this is referred to at the time compression approach. With

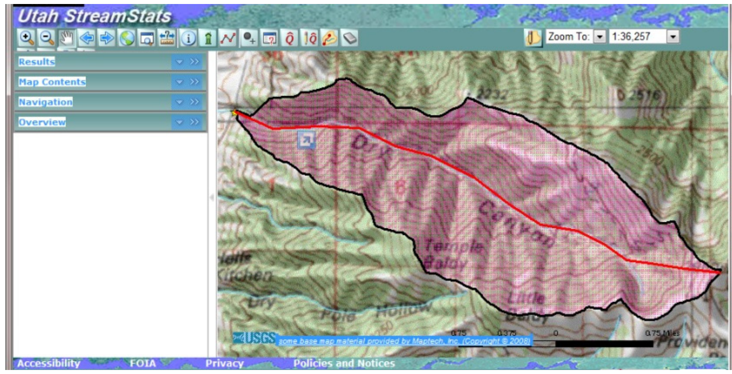
$$F_p = i\,t_p = 0.5616\,h \times 1.5\ cm/h = 0.8424\ cm$$

and t=2 h, F can be solve for numerically (in solver)

F=2.135 cm

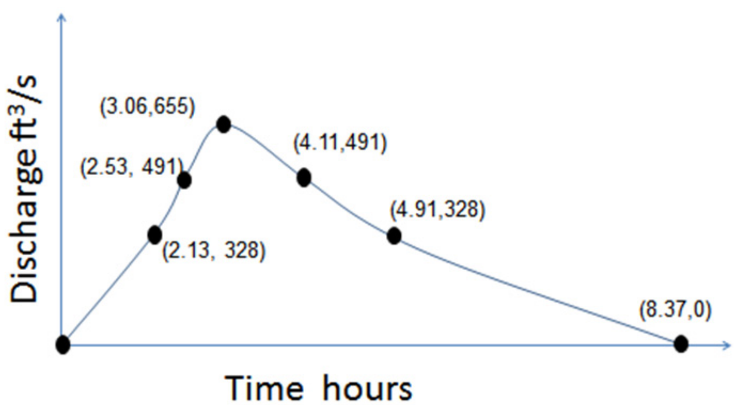
The depth of rainfall is 3 cm, therefore depth of runoff = 3-2.135 = **0.865 cm**.

2. In homework 7 you studied Logan Dry Canyon and determined a Snyder unit hydrograph that is given below



Peak $Q_p = 655\ ft^3/s$

0.5 hour Snyder Unit Hydrograph



Assume a hydrologic soil group C and land use with curve numbers as follows

Forest-range - Herbaceous (fair condition)	40 %	CN=80
Juniper-grass (fair condition)	60%	CN=73

Assume average antecedent moisture conditions. From the NOAA PDFS website (http://hdsc.nws.noaa.gov/hdsc/pfds) the 100 yr 30 min cumulative precipitation is 1.2 in and 60 min cumulative precipitation is 1.49 in. On the basis of these the hyetograph for a design storm is

Time	0-30 min	30-60 min
Rainfall	0.29 in	1.2 in

Determine the following

a) Excess precipitation in each time interval

b) Peak discharge based on the Snyder Unit Hydrograph above

Solution

a) The average curve number is

$CN = 0.4 \times 80 + 0.6 \times 73 = 75.8$ (or 76)

$S = \frac{1000}{CN} - 10 = 3.19\ in$

Initial abstraction $I_a = 0.2\ S = 0.63\ in$

Since in the **first interval** there is only 0.29 in of rainfall it all contributes to initial abstraction, the excess precipitation is 0.

The SCS method is a "whole storm" method and needs to be applied to cumulative rainfall. Cumulatively over the first two intervals the rainfall is 1.49 in. The cumulative excess precipitation is thus

$$P_e = \frac{(P - 0.2\ S)^2}{P + 0.8\ S} = \frac{(1.49 - 0.63)^2}{1.49 + 0.8 \times 3.19} = 0.179\ in$$

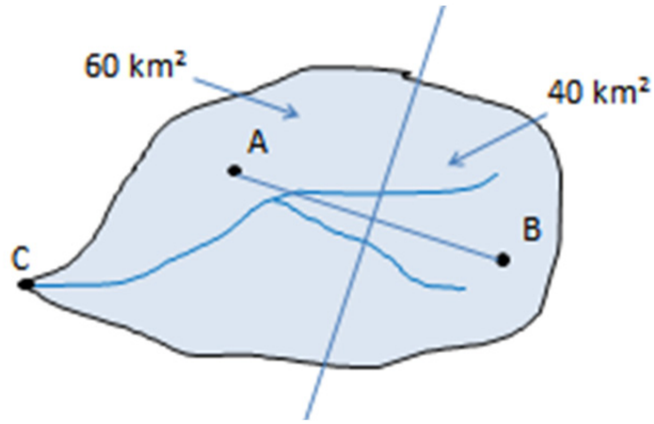
If there had been excess precipitation in the first interval we would need to subtract it from this to determine the 2^{nd} interval P_e. If there was a third interval we would need to compute the cumulative P_e at the end of the third interval and subtract 0.179 in from this to determine the 3^{rd} increment P_e. However in this case, with just two intervals and the first interval excess precipitation 0, we have

Time min	0-30	30-60
Precipitation excess P_e (in)	0	0.179

b) The unit hydrograph is the watersheds unchanging response to a unit of excess precipitation for a fixed duration (the so called duration of the hydrograph). In the hydrograph given, it is a 0.5 h unit hydrograph and we only have one excess precipitation input of 0.179 in. So the hydrograph response to this excess precipitation will be 0.179 × the unit hydrograph values. In particular the peak discharge is $Q_p \times P_e = 655 \times 0.179 =$ **117 ft^3/s**

[If there had been multiple excess precipitation values we would have had to lag and add direct runoff hydrograph components to get the resultant direct runoff hydrograph and determine its peak. But that was not necessary in this case when only the peak was needed]

3. A 100 km^2 total watershed has two precipitation gages in locations indicated



The accumulated rainfall in each gage is given below

Time (min)	A (mm)	B (mm)
0	0	0
30	0	0
60	10	5
90	25	15
120	25	20
150	25	20

An outlet hydrograph measured at location C is

Time (min)	Discharge at C (m^3/s)
0	20
60	20
120	140
180	110
240	70
360	20

Assume a constant baseflow of 20 m^3/s

a) Calculate the area average total precipitation from this storm
b) Calculate the area average precipitation hyetograph for each 30 min increment from this storm
c) Separate the baseflow from direct storm runoff using the assumed constant baseflow and calculate the volume and depth of direct runoff from this storm
d) Assume a constant rate of abstractions and calculate the ϕ-index for this storm
e) Draw a graph of the 30 min excess precipitation hyetograph for this storm

Solution

a) The rainfall data given above is accumulated, or cumulative rainfall. The total rainfall at gage A is therefore the ending value $PT_A = 25$ mm. At gage B it is $PT_B = 20$ mm. The area average total precipitation is obtained by area weighted averaging these values

$$PT = \frac{Area_A \times PT_A + Area_B \times PT_B}{Area_A + Area_B} = \frac{60 \times 25 + 40 \times 20}{60 + 40} = \mathbf{23\ mm}$$

b) The area average precipitation hyetograph for each 30 min increment is obtained similarly

Interval (min)	0-30	30-60	60-90	90-120	120-150
A rainfall increment (mm)	0	10	15	0	0
B rainfall increment (mm)	0	50	10	5	0
0.6 x A + 0.4 x B (mm)	**0**	**8**	**13**	**2**	**0**

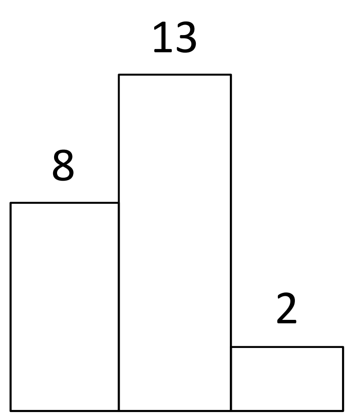
c) The direct runoff is obtained by subtracting the baseflow from the outlet hydrograph

Time (min)	0	60	120	180	240	360
Direct runoff, Q (m^3/s)	0	0	120	90	50	0

The volume is obtained by integrating under this hydrograph.

$$Vol = \Sigma Q\ \Delta t$$
$$\Sigma Q = 260\ m^3/s$$
$$\Delta t = 1\ hr = 3600\ s$$
$$Vol = 260\ \times 3600 = \mathbf{936000\ m^3}$$
$$Depth = \frac{Vol}{Area} = \frac{936000\ m^3}{100 \times 10^6 m^2} = 0.00936\ m = \mathbf{9.36\ mm}$$

d) The area average precipitation hyetograph from (b) is



Assuming constant abstractions of ϕ mm/h the abstraction each 30 min time step is $\phi\ \Delta t$, so the excess precipitation is $P_{ei} = P_i - \phi\ \Delta t$

This only applies for time steps when $P_i > \phi\ \Delta t$, otherwise $P_{ei} = 0$

Now the total depth of runoff equals the sum of P_{ei} over all time intervals it is non zero, i.e.

$$r_d = \Sigma P_{ei} = \Sigma P_i - n\phi\,\Delta t$$

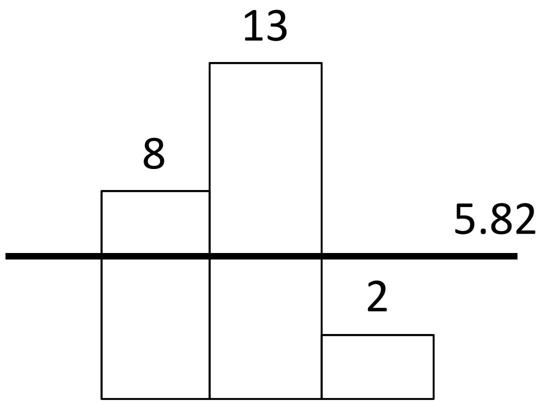
Witn $r_d = 9.36$ mm and $\Sigma P_i = 23$ mm and assuming all time steps are active, i.e. n=3

$$\phi\,\Delta t = \frac{23 - 9.36}{3} = 4.55 \text{ mm}$$

This is greater than the basin average precipitation in the 3rd time interval that is only 2 mm. So remove this from consideration. $\Sigma P_i = 21$ mm, and n=2

$$\phi\,\Delta t = \frac{21 - 9.36}{2} = 5.82 \text{ mm}$$

This is less than the precipitation in the remaining two active time steps so is a valid result



Therefore

$$\phi = \frac{5.82 \text{ mm}}{0.5 \text{ h}} = \mathbf{11.64\ mm/h}$$

e) The excess precipitation hyetograph is obtained by subtracting 5.82 mm from the precipitation in each time interval

Interval (min)	0-30	30-60	60-90	90-120	120-150
Precipitation (mm)	0	8	13	2	0
Excess precipitation (mm)	**0**	**2.18**	**7.18**	**0**	**0**