# CEE3430 Engineering Hydrology

## *Practice Test (There are six practice questions here – A 50 min test will likely not have more than three)*

**Solutions**

1. Frequency Analysis

Following is peak annual flow data from a stream in Utah

| | | | | |
|---|---|---|---|---|
| Mean | 294.7 | | Mean of Logs | 2.389 |
| Variance | 29693 | | Var Logs | 0.0786 |
| Std Dev | 172.3 | | Std Dev Logs | 0.280 |
| Skewness | 0.864 | | Skewness Logs | -0.407 |
| Count | 78 | | **Logs are to base 10** | |

| Q (cfs) | | | | | | |
|---|---|---|---|---|---|---|
| 787 | 436 | 368 | 276 | 224 | 160 | 74 |
| 774 | 436 | 355 | 264 | 206 | 151 | 73 |
| 705 | 432 | 351 | 255 | 201 | 141 | 72 |
| 678 | 429 | 342 | 253 | 195 | 138 | 72 |
| 621 | 424 | 341 | 252 | 193 | 122 | 69 |
| 604 | 415 | 331 | 251 | 184 | 118 | 66 |
| 546 | 408 | 327 | 249 | 179 | 115 | |
| 510 | 404 | 321 | 248 | 178 | 107 | |
| 505 | 398 | 309 | 244 | 174 | 87 | |
| 490 | 391 | 303 | 228 | 172 | 82 | |
| 459 | 381 | 287 | 228 | 170 | 82 | |
| 447 | 372 | 280 | 227 | 164 | 78 | |

a) What is the probability of a flow of 500 cfs being exceeded in any one year

There are 9 flows greater than 500 cfs out of 78 years, so the probability is p=9/78 = **0.115**

b) It is the end of the first year of a 5 year project and the flow of 500 cfs was not exceeded. What is the probability of a flow of 500 cfs being exceeded at least once in years 2 to 5 of the project.

What happened in year 1 is independent and hence not germane, so we are interested in the probability of 500 cfs being exceeded at least once in four sequential years. The probability of non exceedence in 4 years is $(1-p)^4$, so the probability of exceedence at least once in 4 years is $1-(1-p)^4=1-(1-9/78)^4=$**0.388**

c) Assume that this data fits a log-normal distribution, what is the flood with 50 year return period. Comment on whether this is consistent with the data.

For 50 year return period, exceedence probability is 1/T=1/50 = 0.02. Cumulative normal distribution value is F=1-1/T=0.98. Looking up in Normal distribution tables, the standard Z value corresponding to 0.98 is Z=2.054.

50 year return period flow is thus $Q = 10^{(Z\,\sigma_{logs} + \mu_{logs})} = 10^{(2.054 \times 0.28 + 2.389)} = \mathbf{921\ \mathit{cfs}}$

d) Based on the information given is a normal or log-normal distribution likely to be a better fit for this data.

Since the skewness of the logs is closer to 0 than the skewness of the actual data, a log normal distribution fits better. However the difference (skewness of -0.407) is not trivial so in practice a better distribution should be saught. Another indicator of poor fit is the fact that the 50 year return period estimate is larger than the largest flood in 78 years.

2. The relationship between infiltration capacity and cumulative infiltration at a site has been determined from measurements to be given by

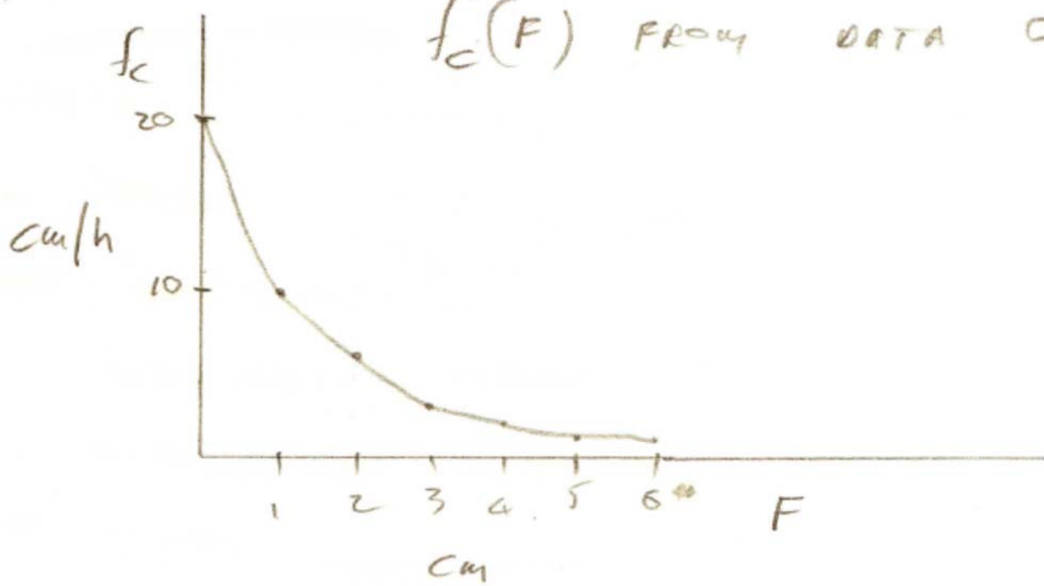
| Cumulative infiltration (cm) | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Infiltration capacity (cm/hr) | 20 | 10 | 6 | 3 | 2 | 1 | 1 | 1 | 1 | 1 | 1 |

Consider a storm in which 12 cm of precipitation falls during 2 hours.

a) Calculate the time to ponding.

a) FUNDAMENTAL RELATIONSHIP

$f_c(F)$ FROM DATA GIVEN




Ponding occurs when $f_c = w = \frac{12}{2}$

$= 6$ cm/h

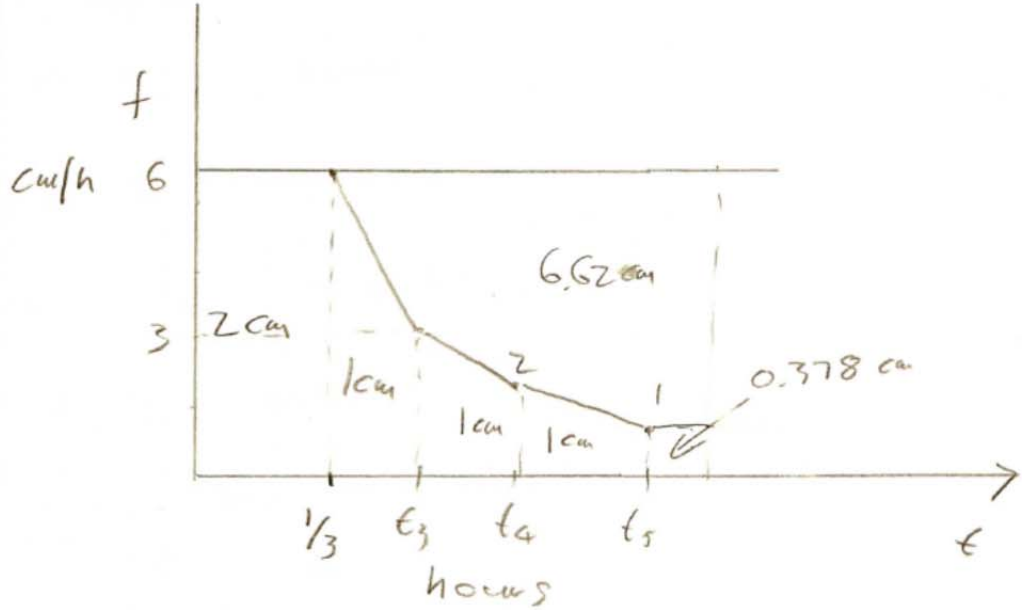
$\therefore \quad F_p = 2$ cm

$\therefore \quad t_p = \frac{2}{6} = \underline{0.333 \text{ hr}}$ TIME TO PONDING

b) Calculate the depth of direct runoff from this storm.

b) By 0.333 = $\frac{1}{3}$ hr in to the storm 2 cm has infiltrated.

Construct infiltration rate with time graph




Let $t_3$ denote the time at which F = 3 cm and $f_c$ = 3 cm/h

The Area of trapezoid from 1/3 to $t_3$ must be 1 cm

$$\therefore \left(t_3 - \frac{1}{3}\right) * \frac{6+3}{2} = 1$$

$$\therefore \quad t_3 = \frac{2}{9} + \frac{1}{3} = \underline{\frac{5}{9}\ hr} = 0.555$$

Let $t_4$ denote the time at which F = 4 cm and $f_c$ = 2 cm/h

Area of trapezoid is again 1 cm

$$(t_4 - t_3) * \frac{3+2}{2} = 1 \Rightarrow t_4 = t_3 + \frac{2}{5} = \underline{0.955\ hr}$$

Let $t_5$ denote time at which $F = 5$ cm
and $f_c = 1$ cm/hr

$$(t_5 - t_4) \times \frac{2+1}{2} = 1$$

$$\therefore \quad t_5 = t_4 + \frac{2}{3} = 1.622 \text{ hr}$$

So after 1.622 hr 5 cm has infiltrated

Rain stops after 2 hr

in time from 1.622 to 2 hr $f = 1$ cm/hr

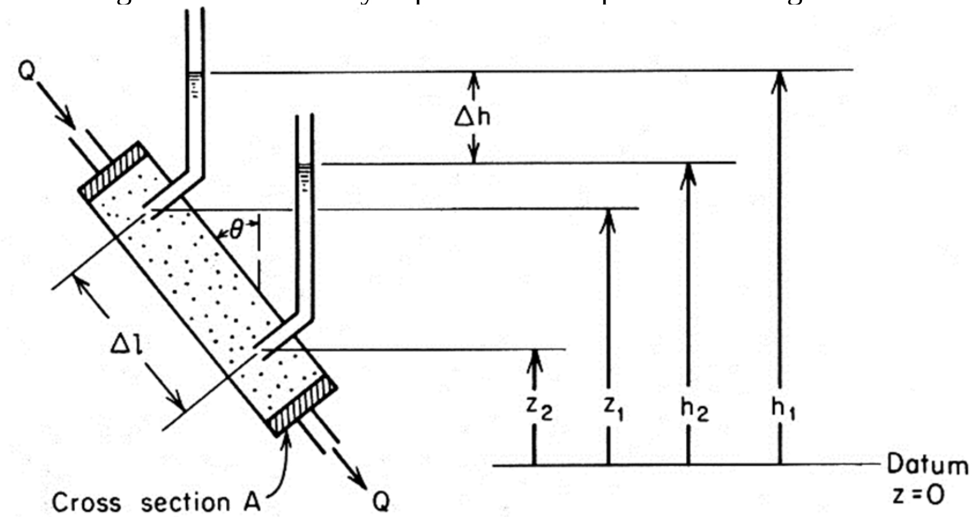
$\therefore$ Infiltration $= 2 - 1.622 = 0.378$ cm

So total infiltration $= 5.378$ cm

$\therefore$ Depth of runoff $= 12 - 5.378$

$= \underline{6.62 \text{ cm}}$

3. Following is data for a Darcy experiment as depicted in the figure.




| | | |
|---|---|---|
| $h_1$ (cm) | 62 | |
| $h_2$ (cm) | 51 | |
| $z_1$ (cm) | 46 | |
| $z_2$ (cm) | 29 | |
| n | 0.45 | Porosity |
| Q ($cm^3$/hr) | 300 | Discharge |
| g (m/s) | 9.81 | Gravitational acceleration |
| $\rho_w$ (kg/$m^3$) | 1000 | Density of water |
| A ($cm^2$) | 50 | Tube internal cross section area |
| Δl (cm) | 40 | Length between piezometers |

a) Calculate the hydraulic gradient

$$\frac{\Delta h}{\Delta l} = \frac{62 - 51}{40} = \frac{11}{40} = \mathbf{0.275}$$

b) Calculate the hydraulic conductivity

$$Q = K\,A\frac{\Delta h}{\Delta l}$$

$$K = \frac{Q}{A\frac{\Delta h}{\Delta l}} = \frac{300\ cm^3/hr}{50cm^2 \times 0.275} = \mathbf{21.8\ cm/hr}$$

4. Consider a soil with the following Green-Ampt infiltration parameters.

| | |
|---|---|
| $K_{sat}$ | 0.6 cm/h |
| $\lvert\psi_f\rvert$ | 20 cm |
| $\Delta\theta$ | 0.2 |

a) Calculate the cumulative infiltration required for ponding and time to ponding for a constant water input rate of 1.5 cm/h.

$$f_c = K_{sat}\left(1 + \frac{|\psi_f|\Delta\theta}{F}\right)$$

At ponding $f_c$=w

$$F_p = \frac{K_{sat}|\psi_f|\Delta\theta}{W - K_{sat}} = \frac{0.6 \times 20 \times 0.2}{1.5 - 0.6} = 2.67\ cm$$

$$t_p = \frac{F_p}{w} = \frac{2.67}{1.5} = 1.78\ hr$$

b) Assume the following storm

| Time (hours) | Rainfall (cm) |
|---|---|
| 0-1 | 1.5 |
| 1-2 | 2 |

Calculate the runoff generated in each 1 hour time interval

In first hour time does not reach $t_p$ for rainfall rate of 1.5 cm/h so there is no runoff

For rainfall rate (water input rate) w = 2 cm/h

$$F_p = \frac{K_{sat}|\psi_f|\Delta\theta}{W - K_{sat}} = \frac{0.6 \times 20 \times 0.2}{2 - 0.6} = 1.7\ cm$$

This represents an additional 0.2 cm infiltration in the second hour so

$$t_p = 1 + \frac{0.2\ cm}{2\,\frac{cm}{hr}} = 1.1\ hr$$

Now solve for infiltration under ponded conditions (equation 49)

$$t - t_s = \frac{F - F_s}{K_{sat}} + \frac{P}{K_{sat}}\ln\left(\frac{F_s + P}{F + P}\right)$$

With $t_s$=$t_p$ = 1.1 hr, t=2 hr $F_s$=$F_p$=1.7 cm, P=20 x 0.2 = 4 cm and $K_{sat}$ = 0.6 cm

Solving implicitly I get F = 3.142 cm, Therefore runoff generated = 3.5 – 3.142 = 0.358 cm. This is all in the second hour.

| Time (hours) | Runoff (cm) |
|---|---|
| 0-1 | 0 |
| 1-2 | 0.358 |

5.

**4.5.** A reservoir has a linear $S$-$Q$ relationship of

$$S = KQ,$$

where $K = 1.21$ hr. The inflow hydrograph for a storm event is given in the table.

a) Develop a simple recursive relation using the continuity equation and $S$-$Q$ relationship for the linear reservoir [i.e., $aQ_2 = bQ_1 + c\bar{I}$, where $a$, $b$, and $c$ are constants and $\bar{I} = (I_1 + I_2)/2$ ].

b) Storage route the hydrograph through the reservoir using $\Delta t = 1$ hr.

c) Explain why the shape of storage-discharge relations is usually not linear for actual reservoirs.

For test

| Time (hr) | Inflow ($m^3/s$) |
|---|---|
| 0 | 0 |
| 1 | 200 |
| 2 | 100 |
| 3 | 0 |

Solution

a) The continuity equation is:

$$\text{In} - \text{Out} = \Delta S/\Delta t$$

Or

$$(I_i + I_{i+1})(\Delta t/2) - (O_i + O_{i+1})(\Delta t/2) = \Delta S = S_{i+1} - S_i$$

Substituting for S = KQ and rearranging yields:

$$(I_i + I_{i+1})(\Delta t/2) - (O_i + O_{i+1})(\Delta t/2) = K\,(O_{i+1} - O_i)$$

$$(K/\Delta t + 0.5)O_{i+1} = I + (K/\Delta t - 0.5)O_i$$

b) For $K = 1.21$ and $\Delta t = 1$, the equation found in part (a) becomes:

$$1.71\ O_{i+1} = I + (0.71)\ O_i$$

| Time (hr) | Inflow ($m^3/s$) | $I=(I_i+I_{i+1})/2$ | Outflow ($m^3/s$) |
|---|---|---|---|
| 0 | 0 | - | 0 |
| 1 | 200 | 100 | (100+0.71*0)/1.71=**58.5** |
| 2 | 100 | 150 | (150+0.71*58.5)/1.71=**112** |
| 3 | 0 | 50 | (50+0.71*112)/1.71=**75.7** |

c) Storage is rarely uniform with depth since few reservoirs are uniform in shape. Most outflow structures have flow relations which are a function of depth raised to a power. Both of these facts lead to a non-linear storage-discharge relation.

6.

**4.6.** Given the reservoir with a storage-discharge relationship governed by the equation

$$S = KQ^{3/2},$$

route the inflow hydrograph for problem 4.5 using storage routing techniques and a value of $K$ = 1.21 for $Q$ in $m^3/s$ and $S$ in $m^3/s$-hr. Discuss the differences in the outflow hydrograph for this reservoir and for the reservoir of problem 4.5. Use $\Delta t$ = 1 hr.

## Solution

From the storage equation given,

$$S = KQ^{3/2} \ , \quad K = 1.21$$

We can obtain the storage-discharge relationship and the storage-indication curve:

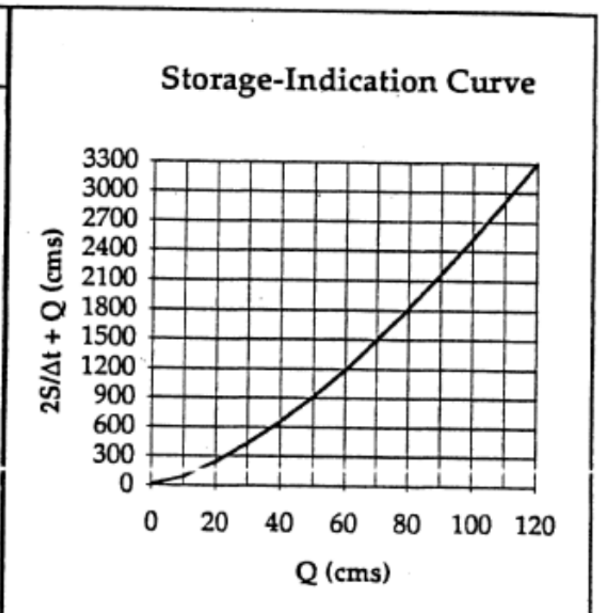
For example, take Q = 50 cms:

$$S = 1.21\ (50)^{3/2} = (1.21)(353.55) = 427.80 \text{ cms-hr}$$

$$2S/\Delta t + Q = [2\ (427.80) \text{ cms-hr} / 1 \text{ hr}] + 50 \text{ cms} = 906 \text{ cms}$$

The following table and graph show these computations for a range of outflows:

| Storage (cms - hr) | Outflow (cms) | 2S/Δt + Q (cms) |
|---|---|---|
| 0.00 | 0 | 0 |
| 38.26 | 10 | 87 |
| 108.23 | 20 | 236 |
| 198.82 | 30 | 428 |
| 306.11 | 40 | 652 |
| 427.80 | 50 | 906 |
| 562.36 | 60 | 1185 |
| 708.65 | 70 | 1487 |
| 865.81 | 80 | 1812 |
| 1033.12 | 90 | 2156 |
| 1210.00 | 100 | 2520 |
| 1395.96 | 110 | 2902 |
| 1590.59 | 120 | 3301 |



The above is from the solutions manual, but you can evaluate fewer values in a test

Puls method routing is based on the equation

$$\frac{(S_{i+1}+S_i)}{\Delta t}=\frac{I_{i+1}+I_i}{2}-\frac{Q_{i+1}+Q_i}{2}$$

which gives

$$\frac{2S_{i+1}}{\Delta t}+Q_{i+1}=I_{i+1}+I_i+\frac{2S_i}{\Delta t}-Q_i$$

| Time (hr) | Inflow I ($m^3/s$) | 2S/Δt-Q | 2S/Δt+Q | Outflow ($m^3/s$) |
|---|---|---|---|---|
| 0 | 0 | 0 (initial value) | | 0 (initial value) |
| 1 | 200 | 3. 200-2*17.6 =164.8 | 1. 200+0+0=200 | 2. **17.6** (interpolating from column to left in the table above) (200-87)/(236-87)*20+ (236-200)/(236-87)*10 |
| 2 | 100 | 6. 464.8-2*31.6 =401.6 | 4. 164.8+200+100 =464.8 | 5. **31.6** by interpolating (464.8-428)/(652-428)*40+ (652-464.8)/(652-428)*30 |
| 3 | 0 | 9. 501.6-2*33.3 =435 | 7. 401.6+100+0 =501.6 | 8. **33.3** by interpolating (501.6-428)/(652-428)*40+ (652-501.6)/(652-428)*30 |
| 4 | 0 | | 10. 435+0+0 =435 | 11. **30.3** by interpolating (435-428)/(652-428)*40+ (652-435)/(652-428)*30 |

The sequence of calculations is shown by the red numbers.