CEE3430 Engineering Hydrology

Practice Exam (There are multiple practice questions here – A 110 min test will likely not have more than four questions)

1. Water Balance

1.11. In a given year, a watershed with an area of 2500 km^2 received 130 cm of precipitation. The average rate of flow measured in a gage at the outlet of the watershed was 30 m^3/sec. Estimate the water losses due to the combined effects of evaporation, transpiration, and infiltration due to groundwater. How much runoff reached the river for the year (in cm)? What is the runoff coefficient?

Solution

Write the water balance as

$$\Delta S = P - Q - ET - Ig$$

Where ΔS = change in storage, P = Precipitation, Q= discharge, ET = Evapotranspiration and Ig=infiltration to groundwater. Assume that ΔS is 0. Then we can write

$$ET + Ig = P - Q$$

Convert Q from m^3/s to depth

$$Q = \frac{30\ m^3}{s} = \frac{30}{2500 \times 10^6} \times 3600 \times 24 \times 365 = 0.378\ m = 37.8\ cm$$

Therefore the combined loss due to ET and infiltration to groundwater is

$$ET + Ig = P - Q = 130 - 37.8 = \mathbf{92.2\ cm}$$

37.8 cm of runoff reached the river.

Runoff coefficient=37.8/130=**0.29**

2. Water Balance

Following is some data I received that pertains to First Dam

Design Volume = 85 acre-ft

Surface Area = 430000 ft^2

a) Calculate the average depth

b) Given a discharge of 1500 ft^3/s in the Logan river calculate how long it would take for the dam to be filled from empty if no flow is released during the filling.

On the basis of your results above comment on whether drawing First dam down in anticipation of a possible Spring Runoff flood due to snowmelt is a feasible flood mitigation strategy.

Solution

a) Average depth = Volume/Area

$$\text{Average Depth} = \frac{85\ acre\ ft \times 43560\ ft^2/acre}{430000\ ft^2} = \mathbf{8.6\ ft}$$

b) Time to fill = Volume/Inflow rate

$$\text{Time to fill} = \frac{85\ acre\ ft \times 43560\ ft^2/acre}{1500\ ft^3/s} = \mathbf{2468\ s = 41\ min}$$

This time to fill is miniscule relative to the expected duration of a spring snowmelt flood, so the dam just does not have the capacity to help reduce flood flows.

3. Evaporation

At a weather station near a lake the following measurements have been reported to you:

Air temperature: 18 ^{o}C
Air pressure: 800 mb
Dew point: 10 ^{o}C
Relative humidity: 60%
Vapor pressure: 32 mb
Net Radiation: 300 cal cm^{-2} day^{-1}
Wind speed: 2.5 m/s at a height of 2 m
Water temperature: 23 ^{o}C

a) Which of these measurements appears to be a discrepancy and why?
b) Calculate the latent heat of vaporization.
c) Calculate the evaporation using the Mass Transfer Method with a=0 and b=0.0118 cm day^{-1} m^{-1} s mb^{-1} in Equation 1-12 or 1-13
d) Calculate the Bowen ratio and the sensible heat transfer
e) Itemize the following terms of the energy balance at the surface of the lake

Q_N Net radiation
Q_h Sensible heat transfer
Q_e Energy used for evaporation (latent heat transfer)
Q_θ-Q_v Net of increase in energy stored in the lake and advected energy of inflow and outflow.
Show the balance of these terms.

Solution

a) At air temperature of 18 C, the saturation vapor pressure from Equation 1-6 is

$$\mathrm{e_{sa}} = 2.7489 \times 10^8 \exp\left(-\frac{4278.6}{T + 242.79}\right) = 2.7489 \times 10^8 \exp\left(-\frac{4278.6}{18 + 242.79}\right) = 20.61\ mb$$

This is less than the reported vapor pressure of 32 mb. Hence a vapor pressure of 32 mb would imply super saturation which is unstable and likely wrong.

b) Latent heat of vaporization from Equatin 1-7 is:

$$\mathrm{L_e} = 597.3 - 0.57\,T = 597.3 - 0.57 \times 23 = 584.2\ cal/g$$

c) Using the Mass Transfer Method

$$\mathrm{E} = bu_2(e_s - e_a)$$

This requires e_s and e_a
At the water temperature

$$\mathrm{e_s} = 2.7489 \times 10^8 \exp\left(-\frac{4278.6}{T + 242.79}\right) = 2.7489 \times 10^8 \exp\left(-\frac{4278.6}{23 + 242.79}\right) = 28.06\ mb$$

$$\mathrm{e_a} = RH \times e_{sa} = 0.6 \times 20.61 = 12.36\ mb$$

Therefore

$$\mathrm{E} = bu_2(e_s - e_a) = 0.0118 \times 2.5 \times (28.06 - 12.36) = \mathbf{0.463\ cm/day}$$

d) The Bowen ratio is (Equation 1-16)

$$R = 0.66 \frac{P}{1000}\left(\frac{T_s - T_a}{e_s - e_a}\right) = 0.66 \frac{800}{1000}\left(\frac{23-18}{28.06-12.36}\right) = \mathbf{0.168}$$

This is the ratio of sensible heat transfer to heat loss by evaporation.

$$R = \frac{Q_h}{Q_e}$$

Now

$$Q_e = E\,\rho\,L_e = 0.463\ cm/day\ \times 1\ g/cm^3 \times 584.2\ cal/g = 270.4\ cal/cm^2/day$$

Therefore the sensible heat transfer is

$$Q_h = R\,Q_e = 0.168\ \times 270.4 = \mathbf{45.5\ cal/cm^2/day}$$

e) Equation 1-14 gives the Energy balance equation pertaining to evaporation

$$Q_\theta - Q_v = Q_N - Q_h - Q_e = 300 - 45.5 - 270.4 = \mathbf{-15.9\ cal/cm^2/day}$$

The terms in the energy balance are

- Q_N Net radiation = 300 cal/cm^2/day (given)
- Q_h Sensible heat transfer = 45.5 cal/cm^2/day
- Q_e Energy used for evaporation (latent heat transfer) = 270.4 cal/cm^2/day
- Q_θ-Q_v Net increase of stored energy … = -15.9 cal/cm^2/day

These balance in the equation $Q_\theta - Q_v = Q_N - Q_h - Q_e$

4. Hydrograph analysis

A watershed is approximately 22 square miles and has the following time-area relationship between its subbasins and travel time to the outlet

Time (hr)	Area (mi^2)
1	7
2	11
3	4

a) Given the following storm calculate the outflow hydrograph and report the peak flow (in ft^3/s) and runoff volume (in ft^3).

Time (hr)	Rainfall Excess (in/hr)
1	0.6
2	1.2

b) Determine the unit hydrograph for this area

c) Assume that development occurs in this watershed such that excess rainfall from the storm is increased to the following

Time (hr)	Rainfall Excess (in/hr)
1	1.0
2	1.7

Calculate the volume (in ft^3) of a detention basin required to hold the increased runoff.

Solution

a) Multiply in/hr x Area to get runoff. To convert to ft^3/s:

1 in/hr over 1 mi^2 in 1 hour = $1/12*5280^2/3600=645.33$ ft^3/s

Time (hr)	Area (mi^2)	Runoff due to 0.6 in/hr in first hour	Runoff due to 1.2 in/hr in second hour (shifted 1 row down to represent lag by 1 hour	Adding runoff from both rainfall increments
1	7	7*0.6*645.33=2710 cfs		2710
2	11	11*0.6*645.33=4259	7*1.2*645.33=5420	4259+5420=9679
3	4	4*0.6*645.33=1549	11*1.2*645.33=8518	1549+8518=10067
4			4*1.2*645.33=3098	3098

The outflow hydrograph is given in the rightmost column. **The peak outflow is 10067 ft^3/s.**
The volume is calculated adding the runoff from both rainfall increments and converting to ft^3
Volume = 2710+9679+10067+3098=25554 ft^3/s hr x 3600 s/hr = **91.9 x 10^6 ft^3**.

b) The unit hydrograph is the response to 1 in/hr of excess rainfall. Similar to the above, this is evaluated as

Time (hr)	Area (mi^2)	Runoff due to 1 in/hr of rainfall
1	7	7*645.33=4517 cfs
2	11	11*645.33=7098
3	4	4*645.33=2581

c) A total of 1-0.6+1.7-1.2 in = 0.9 in of extra rainfall occurs. Over a 22 mi^2 area this results in a volume
$V=22*5280^2*0.9/12=$ **46 x 10^6 ft^3**.

5. Frequency analysis

The demand on a city's water treatment and distribution system is rising to near system capacity because of a long period of hot, dry weather. Rainfall will avert a situation where demand exceeds system capacity. The time between rainfalls in this city at this time of year is described by an exponential distribution (Bedient page 198-199) with an average of five days.
a) Calculate the probability that there will be no rain for the next week (7 days).
b) Calculate the probability that there will be rain at least once in any week.
c) Calculate the probability that there will be two or more weeks (Sunday to Saturday) with no rain in a four week period.

Solution
a) The exponential distribution is given in Bedient, page 198.
$\lambda=1/\text{mean} = 1/5 = 0.2$ $days^{-1}$
The probability of no rain for the next week is equal to the probability that the time to the next storm is greater than 7 days = $1-F(7) = 1-(1-e^{-\lambda 7})=e^{-0.2*7}=$**0.247**
b) The probability of rain at least once in any week = 1-probability of no rain for a week = 1-0.247 = **0.753**
c) The probability of exactly x weeks of n weeks with rain is given by the binomial distribution with p=0.247

$$p(x) = \frac{n!}{x!\,(n-x)!} p^x (1-p)^{n-x}$$

Evaluating

$$p(2) = \frac{4!}{2!\,(4-2)!} 0.247^2 (1-0.247)^{4-2} = 0.208$$

$$p(3) = \frac{4!}{3!\,(4-3)!} 0.247^3 (1-0.247)^{4-3} = 0.045$$

$$p(4) = \frac{4!}{4!\,(4-4)!} 0.247^4 (1-0.247)^{4-4} = 0.003$$

Adding these the probability of two or more weeks with no rain in a 4 week period is 0.208+0.045+0.003 = **0.256**

6. Flood Routing

4.4. An inflow hydrograph is given for a reservoir that has a weir-spillway outflow structure. The flow through the spillway is governed by the equation

$$Q = 3.75\, L y^{3/2} \text{ (cfs)},$$

where L is the length of the weir and y is the height of the water above the spillway crest. The storage in the reservoir is governed by

$$S = 300y \text{ (ac-ft)}.$$

Using Δt = 12 hr, L = 15 ft, and $S_0 = Q_0$ = O, route the inflow hydrograph through the reservoir using the storage indication method.

Time (hours)	Inflow (cfs)
0	0
12	40
24	125
36	10
0	0

Solution.

First develop $\frac{2S}{\Delta t} + Q$ versus Q relationship. $\Delta t = 12 * 3600 = 43200\ s$

y ft	S ft^3 =300y*43560 ft^2/acre	Q (cfs) = 3.75 x 15 x y$^{3/2}$	$\frac{2S}{\Delta t} + Q$ (cfs)
0	0	0	0
1	13068000	56.25	661.25
2	26136000	159.1	1369.1

Now route using the equation

$$\frac{2S_2}{\Delta t} + Q_2 = I_1 + I_2 + \frac{2S_1}{\Delta t} - Q_1$$

Time (hours)	Inflow (cfs)	Q	$\frac{2S}{\Delta t} - Q$	$\frac{2S}{\Delta t} + Q$
0	0	0	0	
12	40	2. 3.4	3. 40-2*3.4=33.2	1. 40
24	125	5. 16.86	6. 198.2-2*16.86=164.48	4. 33.2+40+125=198.2
36	10	8. 25.47	9. 299.48-25.7*2=248.08	7. 164.48+10+125=299.48
0	0	11. 21.95		10. 248.08+10+0=258.08

Order of infilling is shown in red.

For hour 12 $\frac{2S}{\Delta t} + Q = 40$, interpolate Q=40/661.25*56.25

Then evaluate $\frac{2S}{\Delta t} - Q = \frac{2S}{\Delta t} + Q - 2 * Q = 40 - 2 * 3.4 = 33.2$

This is input to the evaluation of $\frac{2S}{\Delta t} + Q$ for hour 24. Continue on following the pattern.

The solution is in the 3rd column of the table above. Note that the peak is considerably reduced from the 125 cfs inflow.

7. Runoff generation

Consider a soil with the following properties pertaining to Philip's Infiltration Equation

Sorptivity, Sp (cm/h$^{0.5}$) 3.0

Conductivity, K_p (cm/h) 0.6

a) Given a precipitation rate of 3 cm/h calculate the cumulative infiltration at ponding and time to ponding.

b) For the following storm calculate the amount of runoff generated in each time step.

Time (hours)	Rainfall rate (cm/h)
0-2	0.5
2-4	3

Solution

a) Equation 62 in Tarboton 2003 module gives

$$F_p = \frac{S_p^2(w - K_p/2)}{2(w - K_p)^2} = \frac{3^2(3 - 0.6/2)}{2(3 - 0.6)^2} = \mathbf{2.11\ cm}$$

b) Follow the flowchart on page 5:21 of Tarboton 2003.

For the Philip model

$$f_c(F) = K_p + \frac{K_p S_p}{\sqrt{S_p^2 + 4K_p F} - S_p}$$

Set up table similar to Table 6 on page 5:38 for results (order of infilling shown in red)

Time	Rainfall Rate	Ft	fc	F'	fc'	Fp or Fs	dt	ts	to	Fend	Infiltration	Runoff
0-2	0.5	1. 0	2. Inf							3. 0.5 cm/h x 2 h = **1 cm**	4. **1 cm**	5. **0 cm**
2-4	3	6. 1cm	7. 5.38 cm	7. 7 cm	8. 1.46 cm/h	9. 2.11 cm	10. 0.37 h	11. 2.37 h	12. 1.98 h	13. 5.47 cm	14. **4.47 cm**	15. **1.53 cm**

Time step 0 to 2 hr. At the beginning of the first time step cumulative infiltration F is 0 cm and hence f_c = infinity. So no ponding at the beginning of the interval.
The rainfall rate is 0.5 cm/h which is less than K_p, the asymptotic infiltration capacity, so ponding can never occur at this rainfall rate and in the first time step all the water infiltrates

2nd time step 2 to 4 hr. F=1 cm from the end of the first time step

$$f_c(F = 1) = K_p + \frac{K_p S_p}{\sqrt{S_p^2 + 4K_p F} - S_p} = 0.6 + \frac{0.6 \times 3}{\sqrt{3^2 + 4 \times 0.6 \times 1} - 3} = \mathbf{5.38\ cm/h}$$

Since this is greater than rainfall rate there is no ponding at the beginning of this time step.
Next we need to determine if ponding starts during this time step. Following the flowchart this is done assuming all water infiltrates F'=1+3 x 2 = 7 cm and evaluating

$$f_c' = f_c(F = 7) = K_p + \frac{K_p S_p}{\sqrt{S_p^2 + 4K_p F} - S_p} = 0.6 + \frac{0.6 \times 3}{\sqrt{3^2 + 4 \times 0.6 \times 7} - 3} = 1.46\ cm/h$$

Since this is less than the rainfall rate, ponding occurs. Alternatively you can immediately recognize from the solution to a) above that F_p for this rainfall rate is less than F' so ponding is going to occur. F_p has already been evaluated for this rainfall rate in a) above.
Increment of time to ponding,

$$dt = \frac{F_p - F_t}{w} = \frac{2.11 - 1\ cm}{3\ cm/h} = 0.37\ h$$

Therefore

$$t_s = 2 + 0.37 = 2.37\ h$$

Time compression offset

$$t_o = t_s - \frac{1}{4K_p^2}\left(\sqrt{S_p^2 + 4K_pF_s} - S_p\right)^2 = 2.37 - \frac{1}{4 \times 0.6^2}\left(\sqrt{3^2 + 4 \times 0.6 \times 2.11} - 3\right)^2$$
$$= 1.98\ h$$

Therefore at t=4 h

$$F = S_p(t - t_o)^{\frac{1}{2}} + K_p(t - t_o) = 3(4 - 1.98)^{\frac{1}{2}} + 0.6(4 - 1.98) = 5.47\ cm$$

Therefore infiltration is F_4-F_2 = 5.47 – 1 = 4.47 cm

Runoff is 2 h x 3 cm/h – 4.47 cm = 1.53 cm.

Results

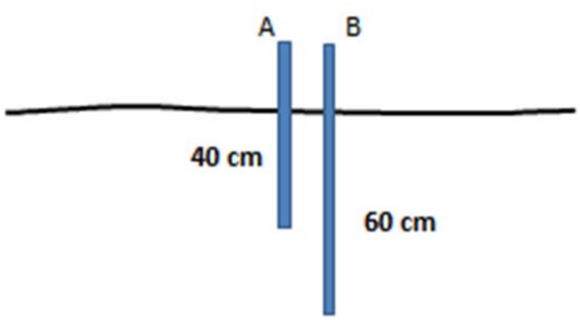
Runoff in first time step: 0 cm

Runoff in second time step: 1.53 cm.

8. Consider the following experimental situation. A and B are vertical tensiometers that measure pore water pressure (tension) relative to atmospheric pressure at depths 40 and 60 cm below the ground. Following are pressure measurements recorded at A and B. Negative denotes capillary suction.

A: -4500 Pa

B: -2000 Pa



a) Evaluate the pressure head (capillary suction) at A and B.

b) Evaluate the total head at A and B using an arbitrary datum 1 m below the ground surface

c) Indicate the direction of flow (downwards into the ground from B to A or upwards from B to A)

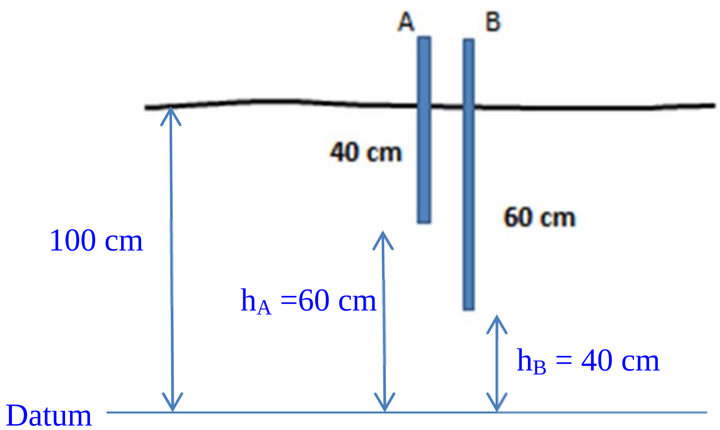
Solution

a) Pressure head

$$\psi_A = \frac{P}{\rho g} = \frac{-4500\ kg\ m\ s^{-2}\ m^{-2}}{1000\ \times 9.81\ kg\ m^{-3}\ m\ s^{-2}} = -0.459\ m = \mathbf{-45.9\ cm}$$

$$\psi_B = \frac{P}{\rho g} = \frac{-2000\ kg\ m\ s^{-2}\ m^{-2}}{1000\ \times 9.81\ kg\ m^{-3}\ m\ s^{-2}} = -0.204\ m = \mathbf{-20.4\ cm}$$

b) Total head



$$h_A = z + \psi_A = 60 - 45.9 = \mathbf{14.1\ cm}$$
$$h_B = z + \psi_A = 40 - 20.4 = \mathbf{19.6\ cm}$$

c) Flow Direction is **upwards from B to A**

9. GroundWater

8.6. In a fully penetrating well, the equilibrium drawdown is 30 ft measured at $r = 100$ ft from the well, which pumps at a rate of 20 gpm. The aquifer is unconfined with $K = 20$ ft/day, and the saturated thickness is 100 ft. What is the steady-state drawdown at the well ($r = 0.5$ ft) for this aquifer?

Solution

From Eq. 8.42

$$Q = \pi K \frac{h_2^2 - h_1^2}{\ln\left(r_2 / r_1\right)} \qquad h_1 = b - s_1^1 = 100\,ft - 30\,ft$$

$$r_1 = 100\,ft$$

$$(20\,gal/\min)\left(\frac{192.5\,ft^3/day}{1\,gal/\min}\right) = \pi(20\,ft/day)\frac{h_2^2 - (100\,ft - 30\,ft)^2}{\ln(-5\,ft/100\,ft)}$$

$$h_2 = 67.64\,ft$$

So the drawdown is 100ft – 67.64ft so $\boxed{s^1 = 32.36\,ft}$ when r = 0.5ft