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The ABC's of snowmelt: a topographically factorized energy component snowmelt model

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Abstract:

Because of the crucial role snowmelt plays in many watersheds around the world, it is important to understand and accurately quantify the melt process. As such, numerous mathematical models attempting to describe and predict snowmelt have arisen. There are two main categories of models: conceptual index models and more intricate energy balance models. The index models, like the degree-day or radiation index models, are practical enough for use in large basins for operational purposes; while the energy balance models, though they are complicated and require large amounts of data, can represent the physics behind melt and give more accurate representations of the spatial distribution of melt within small research basins. The ABC model presented here attempts to bridge the gap between these two extremes by providing a simple yet physically justifiable method that uses elevation and radiation indices together with some measurements to distribute melt over a watershed. This new model separates the energy that causes snowmelt into three components: a spatially uniform component, a component that is proportional to elevation, and one that is proportional to solar illumination (which is determined by topography). Measurements of snowmelt at several topographically unique points (called 'index points') in a watershed are related to elevation and solar illumination through regression in order to factor the melt energy into the three separate components at each time step. The model is driven using inputs from snowmelt measurements at the index locations used to calibrate the regression at each time step. Then the spatial patterns of solar illumination and elevation are used to predict the spatial distribution of melt over the whole watershed. Field data supplemented with synthetically generated data is used to test the model.

KEY WORDS snow; snowmelt runoff; snow hydrology; digital topography; digital elevation data

INTRODUCTION

Snow plays a crucial role in the hydrology of the United States as well as in many other parts of the world. In the western United States, approximately 75% of the total water budget comes from snowmelt (McManamon *et al.*, 1993), and many regions of the world rely heavily on snowmelt for their annual water supply. Of particular interest to hydrologists is the timing and magnitude of melt water fluxes from snow. To this end, numerous models have been suggested and implemented, ranging from simple temperature index melt models to detailed, physically-based models that attempt to accurately represent all of the physical dynamics of snow accumulation, metamorphism, and melt.

In actual engineering practice, the simpler index models, such as the Snowmelt Runoff Model (Martinec *et al.*, 1994), are generally chosen for most jobs. These models have very simple data requirements (usually just a temperature reading, or perhaps measurements of net radiation), and are straightforward to

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implement. They produce reasonable results when well calibrated against prior data for a particular basin, and they are very computationally efficient. These simple index models are practical for general applications where the user needs to know the quantity of runoff expected to appear in the river below. However, they lack sufficient physical basis for many purposes, such as predicting the spatial distribution of melt in heterogeneous terrain or transferring the model to basins with climatic conditions differing from the conditions in the basin for which the model was calibrated.

For these more complicated tasks, numerous detailed energy-balance models have been developed (Anderson, 1976; Flerchinger, 1987; Flerchinger and Saxton, 1989; Jordan, 1991; Tarboton, 1994; Tarboton *et al*., 1995; Tarboton and Luce, 1996). These models can also be used to simulate melt at a larger scale by dividing the area to be modelled into smaller hydrologic units and applying the model separately at each different unit. This gives a relatively good spatial representation of melt. However, a drawback to the energy balance models is that they are generally very data-intensive, requiring either much meteorological instrumentation at the point to be modelled, or interpolation and extrapolation from nearby measurement sites, introducing further uncertainty. Also these models sometimes employ different parameters that need calibration and may even vary throughout the melt season. Finally, these energy-balance models can be very computationally intensive.

A recent focus in snowmelt hydrology has been the development of a model that balances the strengths of the rigorous energy-balance snow models with the strengths of the less data-demanding index models (Kustas *et al*., 1994; Brubaker *et al*., 1996). This paper describes an attempt at such a compromise: the ABC model. This new model is a simple yet physically based method for estimating the spatial distribution of snowmelt based on point snowmelt measurements and topography.

MODEL DESCRIPTION

Most snowmelt models require measurements of weather parameters (such as air temperature, relative humidity, solar radiation, etc.). These models estimate snowmelt by relating these meteorological parameters mathematically to melt using either empirical or physical relationships. Here we take a different approach with the ABC model. Rather than using meteorological measurements, the ABC model is driven using input from a few measurements of actual snowmelt within a basin. The model then extrapolates that melt to the rest of the basin according to topography.

The ABC model is based on the fact that snowmelt is an energy-driven process and that the energy available for melt is primarily dependent upon solar radiation and air temperature, which are both functionally related to topography. Solar radiation is a function of slope, aspect, and shading, while air temperature is commonly considered to be a function of elevation (Dingman, 1994). The assumption is therefore made that the spatial distribution of energy can be partitioned into components that depend on a radiation factor and an elevation factor, as well as a spatially constant component (based on the derivation given later). The ABC model can be stated mathematically as follows:

$$\Delta h_{mi} = \max[(A(t) + B(t) \cdot elev_i + rad_i \cdot C(t)), 0] \quad (1)$$

where Δh_{mi} is the depth of melt that occurs over the time step at location i expressed in snow water equivalent; rad_i is the direct, exoatmospheric radiation at location i determined from the slope, aspect and shading due to nearby terrain and integrated over the timestep; and $elev_i$ is the elevation of location i. The terms $A(t)$, $B(t)$, and $C(t)$ (hence the name ABC model) represent a time-dependent factorization of the melt-producing energy during the time step. $A(t)$ is a topographically independent component quantifying the base melt rate in the watershed caused by the weather conditions during the current timestep, $B(t)$ is an elevation-dependent component quantifying the effect of elevation (as a surrogate for temperature) upon melt, and $C(t)$ is a radiation-dependent component describing the transmissivity of the atmosphere to incoming solar radiation as well as the albedo of the snow. These energy factors, $A(t)$, $B(t)$, and $C(t)$, are assumed spatially constant throughout the whole watershed for each time step, but they do change from one time step to the next, depending on the melt-producing effect of each of the energy terms integrated over the time step.

Ignoring the maximum operator in equation (1) gives a linear equation with three unknown variables at each time step: $A(t)$, $B(t)$, and $C(t)$. The terms $elev_i$ and rad_i can be easily determined for every point in the watershed. Therefore, given sufficient melt observations at topographically unique points in the watershed, regression can be used to estimate effective values for $A(t)$, $B(t)$, and $C(t)$. If the maximum operator is included in equation (1), one can solve for the $A(t)$, $B(t)$, and $C(t)$ parameters by minimizing the error when fitting the equation to the observations. Once these three variables are established for the given time step, the melt that occurs at every point in the entire basin can be quickly calculated through equation (1).

Operational overview

Figure 1 shows a flowchart for the ABC model. The model requires a digital elevation model (DEM) of the watershed as shown in the upper-left corner of the flowchart. From this DEM, elevations, slopes, and aspects are calculated for each grid cell in the watershed. The slopes and aspects are used to calculate the amount of exoatmospheric radiation that each point in the watershed receives during a given time step.

Next, Figure 1 shows that the ABC model requires measurements of melt at a number of topographically unique locations throughout the watershed. These measurement sites are referred to as 'index points'. A sufficient number of melt measurements must be collected within the watershed to obtain a sound calibration of equation (1) at each time step. The melt at these index points would ideally be measured with automated melt collectors, and the data relayed back to a central processing station in real time. Once the central processing station has received the values for total melt occurring at each of the sampling locations over a specified time period, the model then calculates the values of $A(t)$, $B(t)$, and $C(t)$ in real time for each time period. Then, using the elevations from the DEM and the calculated radiation indices, the melt is estimated from equation (1) for all unmeasured points in the watershed that are still covered with snow, as indicated by the current map of snow-covered area.

Finally, Figure 1 shows that these estimates of melt may be used as inputs to a flow-routing module, which is used to generate a prediction for streamflow. This is usually the quantity of interest.

THEORETICAL DERIVATION OF THE ABC MODEL

This section shows how to obtain equation (1) from the physical energy balance equations relating driving meteorological variables to snowmelt. The basic theory behind all of the physically-based point snowmelt models lies in balancing the energy budget for the snowpack and converting the excess energy into snowpack temperature change, metamorphism, or melt. The melt period of a seasonal snowpack begins when the net energy input starts to have a positive trend. This period can be separated into the warming phase, the ripening phase, and the output phase (Dingman, 1994).

During the warming phase, the net energy input raises the temperature of the snowpack until the whole pack reaches the melting point, as such:

$$\Delta Q = c_i \rho_w h_m \Delta T_{avg} \tag{2}$$

where ΔQ is the total positive energy input to the snowpack during a given time interval ($\mathrm{J \cdot cm^{-2}}$), c_i is the heat capacity of ice ($2{\cdot}102\ \mathrm{J \cdot g^{-1}\,{}^{\circ}C^{-1}}$), ρ_w is the density of water ($1\ \mathrm{g \cdot cm^{-3}}$), h_m is the depth of water equivalent of the snowpack (cm), and ΔT_{avg} is the average change in temperature of the snowpack.

During the ripening phase and the output phase, the snowpack remains isothermal at the melting point, but the additional energy input causes some of the snow to change phase from ice to water according to the following equation:

$$\Delta Q = \lambda_f \rho_w \Delta h_m \tag{3}$$

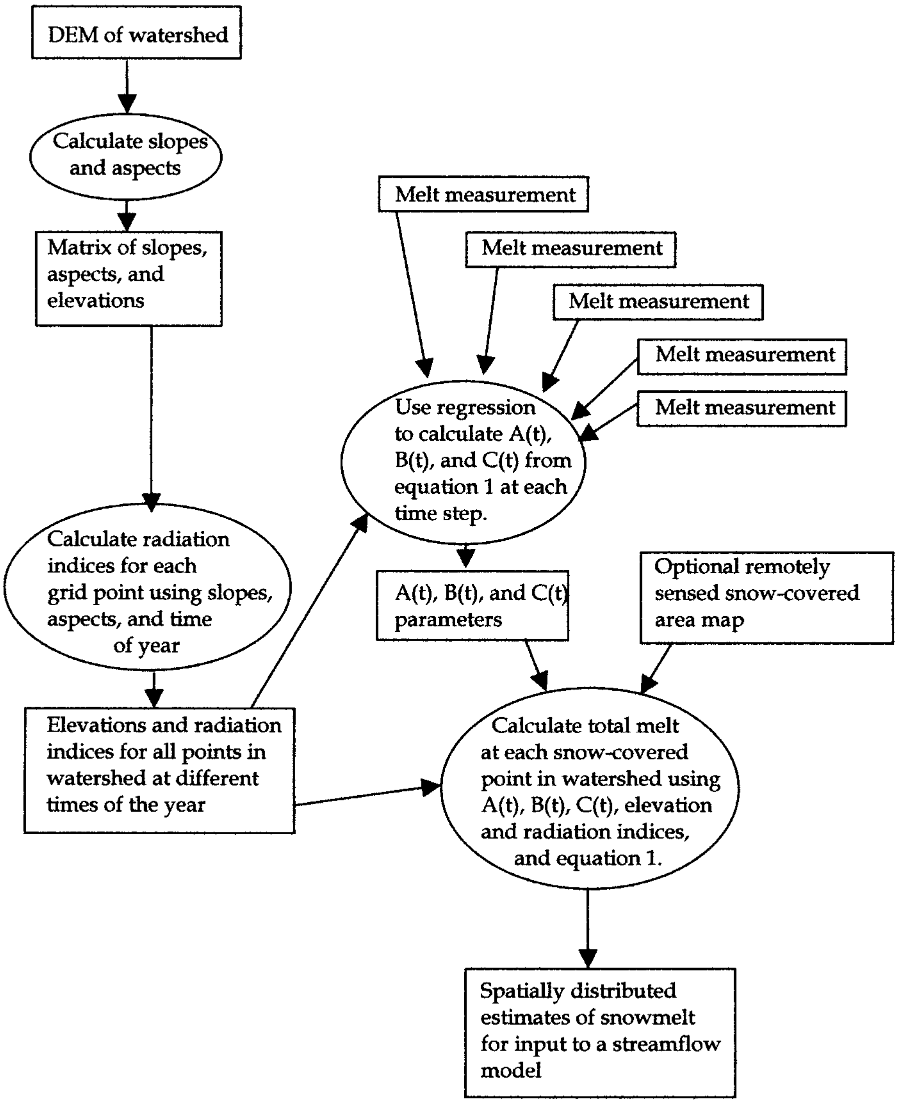

Figure 1. Operational flowchart for the ABC model

where λ_f is the heat of fusion of ice at 0 °C (333·7 J · g^{-1}) and Δh_m is the depth of snow in water equivalence that is converted to water (cm). During the ripening phase, the liquid water is retained in the snowpack by surface-tension forces until the snow reaches its liquid holding capacity. After this, the output phase begins and melt water flows out of the snowpack. Consequently, during the output phase (which is the phase of concern for this model) the energy input is directly proportional to the amount of melt outflow.

The energy balance equation relating the meteorological driving forces to snowmelt is:

$$\Delta Q = Q_{sn} + Q_{ln} + Q_p + Q_g + Q_h + Q_e \tag{4}$$

where Q_{sn} is the net shortwave energy received by the snowpack, Q_{ln} is the net longwave radiation into the snow, Q_p is the energy advected by precipitation into the snow, Q_g is the ground heat flux to the snow, Q_h is the sensible heat flux to the snow, and Q_e is the latent heat flux into the snow (Dingman, 1994).

Physically based melt models usually require measurements of driving weather variables such as air temperature, solar radiation, wind speed, etc. These variables are used as inputs for various equations to determine the quantities of the different components of the energy balance, and the sum of these energy components is used to estimate the melt occurring in a snowpack. One obstacle to this method of physically modelling snowmelt is the difficulty of accurately measuring these driving variables and then appropriately using them to calculate each of the terms in the energy balance (equation (4)).

The ABC model differs from most other melt models in that it uses direct, real-time measurements of melt to drive the model, rather than relying on measurements of the driving weather variables and trying to relate these back to melt rates. As such, it uses several spatially distributed measurements of actual snowmelt to effectively back-calculate the driving energy terms. These inferred energy inputs are representative of how the weather in the watershed has actually affected snowmelt. These calculated energy components are then used to calculate the melt for the rest of the watershed.

The theoretical justification for this model (equation (1)) lies in recognizing melt as a linear function of available energy, and then approximating each of the components of the energy balance equation (equation (4)) as linear functions of elevation and potential solar illumination. In order to derive equation (1), all of the energy fluxes must be expressed as linear functions of solar illumination and air temperature (and thus elevation, assuming that temperature varies linearly with elevation).

Net solar radiation at any point, *i*, is comprised of diffuse and direct components with the direct component related to illumination angle (the angle of the sun from the perpendicular to the land surface) as follows:

$$Q_{sni} = Q_{sndif} + \int_{\Delta t} I_0 \cdot \tau \cdot (1 - Alb) \cdot \cos(\psi) \cdot dt \tag{5}$$

where Q_{sni} is the net shortwave radiation received at point i over the interval Δt, Q_{sndif} is the net diffuse shortwave radiation received during the timestep, I_0 is the exoatmospheric solar radiation constant, τ is the atmospheric transmissivity to direct beam radiation, Alb is the albedo, and ψ is the illumination angle. Now, neglecting the fact that Alb depends upon illumination angle and changes slowly with time as the surface ice crystals grow, and τ changes with weather and sun angle, Alb and τ are taken out of the integral, and the integral of $I_0 \cdot \cos(\psi)$ at location i is expressed as rad_i. Assuming that the atmospheric transmissivity, τ, and the albedo of the snow surface, Alb, are unknown but spatially uniform throughout the watershed (justification for and implications of this assumption are discussed at the end of this section), the above equation for net solar radiation can be written as:

$$Q_{sni} = rad_i \cdot C(t) + Q_{sndif} \tag{6}$$

where τ and $(1 - Alb)$ are combined into the factor $C(t)$. Dingman (1994) gives a formula for the calculation of rad_i for a given slope, aspect, and latitude at a daily time step. Dozier and Frew (1990) have presented TOPORAD, a model that can rapidly compute rad_i using digital elevation data incorporating the effects of terrain shading in its calculations.

Net longwave radiation can be expressed as:

$$Q_{ln} = \varepsilon_a \sigma \cdot Ta^4 - \varepsilon_{ss} \sigma Ts^4 \tag{7}$$

where ε_a is the effective emissivity of the atmosphere, σ is the Stefan-Boltzmann constant, Ta is the air temperature in the basin for the given time period, ε_{ss} is the emissivity of the snow surface, and Ts is the temperature of the snow surface (all temperatures here are relative to absolute zero). Since it is desirable to write the energy fluxes as linear functions of air temperature, the above equation is approximated as a truncation of a first-order Taylor series expansion about Ta_{ref}, a constant reference temperature as follows:

$$Q_{ln} \cong \varepsilon_a \sigma \cdot Ta_{ref}^4 - \varepsilon_{ss} \sigma Ts^4 + 4\varepsilon_a \sigma \cdot Ta_{ref}^3 \cdot (Ta - Ta_{ref}) \tag{8}$$

Assuming that ε_a, ε_{ss} and Ts are uniform throughout the basin (again, the impacts of these assumptions are discussed at the end of this section), the terms in the last equation can be condensed to the following linear function of air temperature:

$$Q_{ln} = A_{ln} + B_{ln} \cdot Ta \tag{9}$$

Under conditions of neutral buoyancy, turbulent mass transfer theory (Dingman, 1994) gives the sensible heat flux, Q_h, as:

$$Q_h = \frac{k^2 V}{\left(ln\frac{z}{z_0}\right)^2} \rho_a C_p \cdot (Ta - Ts) \tag{10}$$

where k is the von-Karman constant, V represents the wind speed, z is the height at which the wind speed is measured, z_0 denotes the effective aerodynamic roughness of the snow surface, ρ_a is the density of the air, and C_p represents the specific heat capacity of air. Thus, sensible heat flux is already a linear function of air temperature, and the above equation can be simplified to:

$$Q_h = A_h + B_h \cdot Ta \tag{11}$$

This simplification has lumped all of the variability in quantities such as wind speed (V), roughness height (z_0), air density (ρ_a), and the surface temperature (Ts) into the parameters A_h and B_h, neglecting their spatial variability. Of these, the spatial variability in wind speed is perhaps the most serious; however, it is difficult to quantify this variability in a simple way. The air density also varies since it is related to elevation. The impact on the model results caused by the spatial variability of these parameters is discussed at the end of this section.

Turbulent mass transfer theory (Dingman, 1994) gives the latent heat flux to the surface, Q_e, as:

$$Q_e = \frac{k^2 V}{\left(ln\frac{z}{z_0}\right)^2} \frac{h_v 0{\cdot}622}{R_d Ta} (e_a - e_s) \tag{12}$$

where h_v equals the latent heat of vaporization for water, R_d is the dry gas constant, and e_a and e_s are the vapour pressures of the air and surface respectively. These are related to temperature, relative humidity, RH, and the saturation vapour pressure versus temperature function $e_{sat}(T)$ by:

$$e_a = e_{sat}(Ta) \cdot RH \tag{13}$$

$$e_s = e_{sat}(Ts) \tag{14}$$

Lowe (1977) provides accurate polynomial expressions for $e_{sat}(T)$.

Again, taking a first-order truncation of the Taylor's series expansion of this equation about a reference temperature, Ta_{ref}, gives the following expression:

$$Q_e \cong \frac{k^2 V}{\left(ln\frac{z}{z_0}\right)^2} \frac{h_v 0{\cdot}622}{R_d Ta_{ref}} [RH \cdot e_{sat}(Ta_{ref}) - e_{sat}(Ts)] + \frac{k^2 V}{\left(ln\frac{z}{z_0}\right)^2} \frac{h_v 0{\cdot}622}{R_d} \left(\frac{RH \cdot \Delta}{Ta_{ref}} - \frac{RH \cdot e_{sat}(Ta_{ref})}{Ta_{ref}^2} + \frac{e_s(Ts)}{Ta_{ref}^2}\right)(Ta - Ta_{ref}) \tag{15}$$

where Δ represents the derivative of e_{sat} with respect to air temperature evaluated at Ta_{ref}.

Assuming that V, z_0, and RH are uniform throughout the watershed for a given time period (RH will change with elevation, but again this is rationalized at the end of this section), the terms in the above equation can be condensed to the following:

$$Q_e = A_e + B_e \cdot Ta \tag{16}$$

Equations (4), (6), (9), (11) and (16) can now be combined to write the following equation:

$$\Delta Q = rad_i \cdot C(t) + Q_{sndif} + A_{ln} + B_{ln} \cdot Ta + Q_p + Q_g + A_h + B_h \cdot Ta + A_e + B_e \cdot Ta \tag{17}$$

Though the terms $C(t)$, Q_{sndif}, A_{ln}, B_{ln}, Q_p, Q_g, A_h, B_h, A_e, and B_e may vary with time, they are (by approximation) spatially constant throughout the watershed. By condensing these terms into single constants (A_{comb} and B_{comb}), the above equation can be simplified to:

$$\Delta Q = rad_i \cdot C(t) + A_{comb} + B_{comb} \cdot Ta \tag{18}$$

Next, Ta is assumed to vary linearly with elevation according to an unknown lapse rate as such:

$$Ta = a \cdot elev_i + b \tag{19}$$

Combining equations (3), (18), and (19) gives:

$$\lambda_f \rho_w \Delta h_m = rad_i \cdot C(t) + A_{comb} + B_{comb}[a \cdot elev_i + b] \tag{20}$$

Finally, condensing the constants in the above equation gives an expression for Δh_m, which is the depth of snowmelt in water equivalent:

$$\Delta h_m = rad_i \cdot C(t) + A(t) + B(t) \cdot elev_i \tag{21}$$

Equation (21) assumes a ripe snowpack (equation (3)), and as such is only valid for positive net energy contributions, i.e. $\Delta h_m > 0$. Therefore, adding this necessary condition that Δh_m is the greatest of either the energy input or 0 (since negative energy does not result in negative melt) results in equation (1):

$$\Delta h_{mi} = max[(A(t) + B(t) \cdot elev_i + rad_i \cdot C(t)), 0] \tag{22}$$

where the subscript i denotes the location. Here $A(t)$ is an unknown energy input factor that is a function of time and is representative of the base energy input to the watershed, equal to the following:

$$A(t) = (A_{comb} + B_{comb} \cdot b)/\rho_w \lambda_f \tag{23}$$

$B(t)$ is another unknown energy input factor that varies with time and represents the effect of elevation (or air temperature) upon snowmelt. It is equal to:

$$B(t) = B_{comb} \cdot a/\rho_w \lambda_f \tag{24}$$

Finally, $C(t)$ is a time-dependent factor that takes the net incoming direct shortwave radiation into account as explained above. These three energy input factors vary with time, but at the end of each time step, they are assumed to be the same for every point in the whole watershed.

Because of the maximum operator, equation (1) is a non-linear equation with three unknown variables, $A(t)$, $B(t)$, and $C(t)$. The terms rad_i and $elev_i$ are known quantities for each point in the watershed. Therefore, given melt observations at several points in the watershed (index points), one can fit the equation to the data by minimizing either the sum of absolute errors or the sum of squared errors, and obtain unique values for the parameters $A(t)$, $B(t)$, and $C(t)$. Once these three variables are established, the melt that occurs at every point in the entire basin can be calculated through equation (1). This procedure can be used at every timestep for which there are new measurements of melt to determine the amount of melt that occurred throughout the basin during that timestep.

It is interesting to note that the three terms in equation (1) primarily describe the effects of the terrain on net radiation and turbulent transfer. Consistently in the literature, these two energy sources have been shown to dominate the snowmelt process, to the point where the other terms are usually negligible. Cline (1997) described a thorough experiment in the mountains of Colorado that quantified all of the terms of the energy balance during two melt seasons. He found that net radiation and turbulent fluxes were of the greatest importance, but that their relative contributions to the overall energy balance varied from year to year. Kuusisto (1986) reviewed over 20 studies of snowmelt energy balances and came to the similar conclusion that net radiation and turbulent fluxes were the dominant energy components driving snowmelt.

Equation (1) clearly models only melt, not the spatial distribution of snow water equivalence. With the restriction that $\Delta h_m > 0$, equation (1) is incapable of modelling the accumulation of snow. However, if desired, the ABC model can be altered to keep track of the amount of water equivalence in the snowpack. In this case, the snow water equivalence needs to be maintained for each point as a state variable. The formula governing the change in snow water equivalence at each point i is:

$$\Delta w_i = precip_i - melt_i \tag{25}$$

Assuming that precipitation is approximately linearly related to elevation (Dingman *et al.*, 1988), we can write the following equation:

$$precip_i = \varphi(t) \cdot elev_i + \kappa(t) \tag{26}$$

where $\varphi(t)$ is the unknown lapse rate for precipitation and $\kappa(t)$ is the unknown base snowfall rate for the basin during the given time period.

Computing $melt_i$ as $max[(A(t) + B(t) \cdot elev_i + rad_i \cdot C(t)), 0]$, as given in equation (1), results in the following expression for snow water equivalence:

$$\Delta w_i = \varphi(t) \cdot elev_i + \kappa(t) - max[(A(t) + B(t) \cdot elev_i + rad_i \cdot C(t)), 0] \tag{27}$$

This equation assumes that the advected energy from the precipitation has a negligible effect on the energy balance of the snowpack. This is a reasonable assumption in most circumstances. In his review of over 20 snowmelt energy balance studies, Kuusisto (1986) found that on average, energy advected from precipitation accounts for less than 1% of the energy budget.

If snow does indeed fall during a time step, the above equation can be simplified by dropping the maximum operator requiring that melt be greater than zero and combining the similar terms to give:

$$\Delta w_i = A_{swe}(t) + B_{swe}(t) \cdot elev_i + rad_i \cdot C(t) \tag{28}$$

where

$$A_{swe}(t) = A(t) + \kappa(t) \tag{29}$$

$$B_{swe}(t) = B(t) + \varphi(t) \tag{30}$$

The ABC model for snow water equivalence can now be stated as follows:
If there is no precipitation during a timestep:

$$\Delta w_i = max[(A_{swe}(t) + B_{swe}(t) \cdot elev_i + rad_i \cdot C(t)), 0]$$

If there is precipitation during a timestep:

$$\Delta w_i = A_{swe}(t) + B_{swe}(t) \cdot elev_i + rad_i \cdot C(t) \tag{31}$$

The first part of equation (31) is simply equation (1), while the second part of equation (31) is a linear equation whose unknown variables can be solved for using simple linear regression. Therefore, given several measurements of the change in water equivalence at different points in the snowpack, one can use equation (31) to model the snow water equivalence just as one would use equation (1) to model melt.

Finally, let us examine some of the original assumptions more carefully. In the derivation of equation (1) and subsequently equation (31), the atmospheric transmissivity (τ), albedo of the snow surface (A), snow surface temperature (Ts), relative humidity (RH), air density (ρ_a), and wind speed (V), were all assumed to be spatially invariant across the watershed. The air density, atmospheric transmissivity, and the relative humidity will change across the watershed, but the gradients will probably be very strongly correlated with elevation. Therefore, the choice of elevation as a parameter in the model will incorporate most of these effects, though only in a linearized fashion. The albedo of the snow surface and the surface temperature will most likely vary in space; however, these parameters are intimitely related to the energy that the snowpack has received, which is primarily determined by the amount of radiation received and the turbulent heat exchange that has occurred over the snowpack. Therefore, the model's dependence on elevation and radiation will incorporate the effects of changing albedo and surface temperature, though again, only in a linearized fashion. Finally, though these assumptions are not completely rigorous, they are all justified as pragmatic and expedient in terms of the quality of the resulting approximations, as demonstrated in the next section.

MODEL TESTING

This section provides an analysis of the performance of the ABC model with two sets of data. The first data set was synthetically generated by another snowmelt model. The second set of data comes from field measurements taken in Smithfield Dry Canyon in the spring of 1997.

Because of the potential for measurement error when collecting snowmelt data, robust least-absolute-error regression was used to calculate the $A(t)$, $B(t)$, and $C(t)$ parameters in all the results presented.

Synthetically generated data

Because of limited field data, much of this research was performed with data that was synthetically generated by the Utah Energy Balance (UEB) model (Tarboton and Luce, 1996; Tarboton *et al.*, 1995). The UEB model is a physically based energy balance model that represents the snowpack in terms of two state variables; water equivalence and energy content. A third state variable is used to quantify the snow surface age, which is used for albedo calculations. The use of only three state variables makes the model less complicated than typical energy balance melt models and therefore suitable for generating many simulations of melt at different points. The model uses a parameterization of surface heat flux into the snow based on the difference between the snow surface and average snowpack temperatures to balance external energy fluxes at the snow surface and to calculate snow surface temperature without introducing additional state variables.

In order to generate a full set of snowmelt data with which to test the ABC model, a hypothetical terrain with 200 points was simulated. Each of these 200 points were assigned slopes randomly selected from a uniform distribution over the range of 0° to 60°, random aspects ranging uniformly all the way around the compass, and random elevations selected from a uniform distribution over a range of 1500 m. Next, the UEB model was used with weather data recorded at the Central Sierra Snow Laboratory during the winter of 1986–1987 to calculate snow accumulation and melt at each of the hypothetical points. The incoming exoatmospheric radiation at each point was modified to account for slope and aspect, while the air temperature for each point was adjusted according to a lapse rate of 1 °C for every 150 m that the point was above the datum (Dingman, 1994). In this manner, a time series of snowmelt was generated for every point in the hypothetical watershed. The simulated snow water equivalence (SWE) at every point was recorded every 48 hours, and the melt during each 48-hour period was calculated as the difference in SWE between subsequent time steps. Five index points within the hypothetical basin were deemed sufficient to determine the $A(t)$, $B(t)$, and $C(t)$ terms. The following points were chosen as the index points to be used throughout the melt season:

	elevation:	slope:	aspect. clockwise from North:	characteristics:
point 1	2650 m	57°	316°	low radiation, med. elevation
point 2	3420 m	59°	353°	low radiation, high elevation
point 3	2166 m	54°	169°	high radiation, low elevation
point 4	3390 m	56°	72°	med. radiation, high elevation
point 5	2869 m	58°	116°	high radiation, med. elevation

These points are extremely varied in terms of elevation and radiation received. This wide variation in topography of index points provides stability in the regression equation and is important for accurate melt predictions.

Figure 2 shows eight plots of the ABC model estimates versus UEB-generated melt for each of the 200 points. Only those points at which the UEB model still predicts the presence of snow are shown, since the results are meaningless if the snow has completely melted. These correspond to the eight 48-hour melt periods from 28 March to 13 April. The solid line in each plot is the 1 : 1 line, representing a perfect fit. As can be seen, the ABC model performed fairly well, but not perfectly. The adjusted multiple coefficients of determination (or R^2 values) for each plot are given in the figure. Following Mendenhall and Sincich (1992), these R^2 values were calculated as follows:

$$R^2 = 1 - \frac{(n-1)}{n-(k+1)}\left(\frac{\Sigma(y-\hat{y})^2}{\Sigma(y-\bar{y})^2}\right) \tag{32}$$

where y represents the values of melt predicted by the UEB model (or 'measured' melt value), $\hat{y}$ represents the values of melt predicted by the ABC model, n is the total number of predictions, and k is the number of parameters used in the regression (three).

In Figure 2, it is apparent that the linear ABC model is not entirely appropriate for modelling this synthetic data set. Though most points fall on or near the 1 : 1 lines, the points on the left-hand sides of the graphs cluster into vertical lines, indicating that melt is not a continuously linear function of radiation and elevation near the points with zero melt. This is the region where 'melt' is negative and actually represents accumulation. There is precipitation during these intervals so the second form of equation (31) has been used. This is somewhat artificial because the ABC model tries to relate snow accumulation to elevation using the same linear function as was used for melt at lower elevations, while in the UEB model simulations precipitation was held constant across the basin. In reality we expect some variation of precipitation with elevation.

Figure 3 shows the time series of snow accumulation and melt for eight sites that were selected semi-randomly to show different extremes of accumulation and ablation. The points show the actual UEB results, while the lines show the ABC estimates of the melt process using equation (31). The average adjusted multiple coefficient of determination (R^2) for the eight points shown is 0·95, while the average R^2 for all of the 200 points is 0·94.

Field data

During the 1997 melt season, melt measurements were collected at 31 locations in Smithfield Dry Canyon at approximately five-day intervals. These locations were distributed over a range of 1200 feet (366 m) in

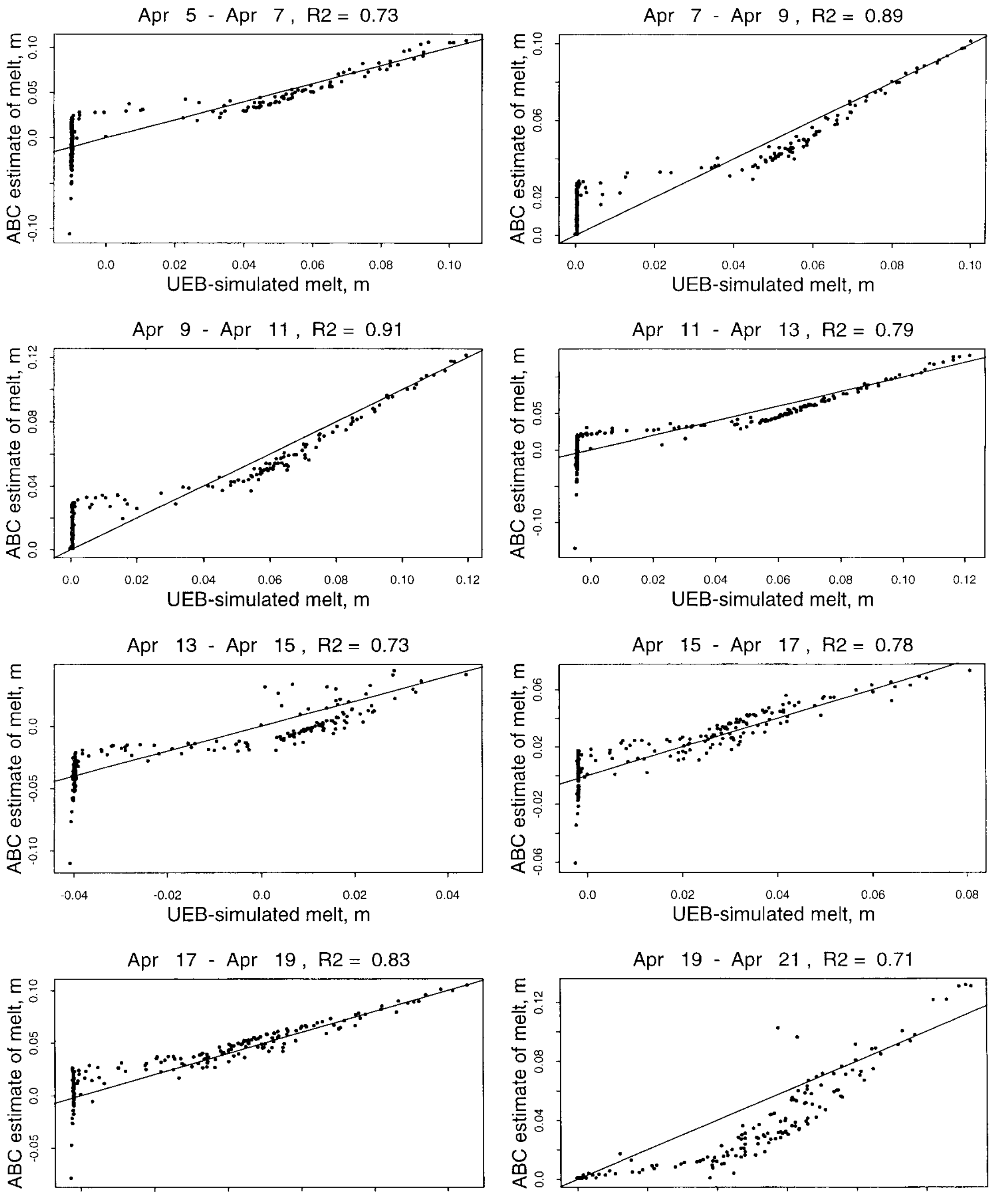

Figure 2. ABC model estimates versus UEB simulations

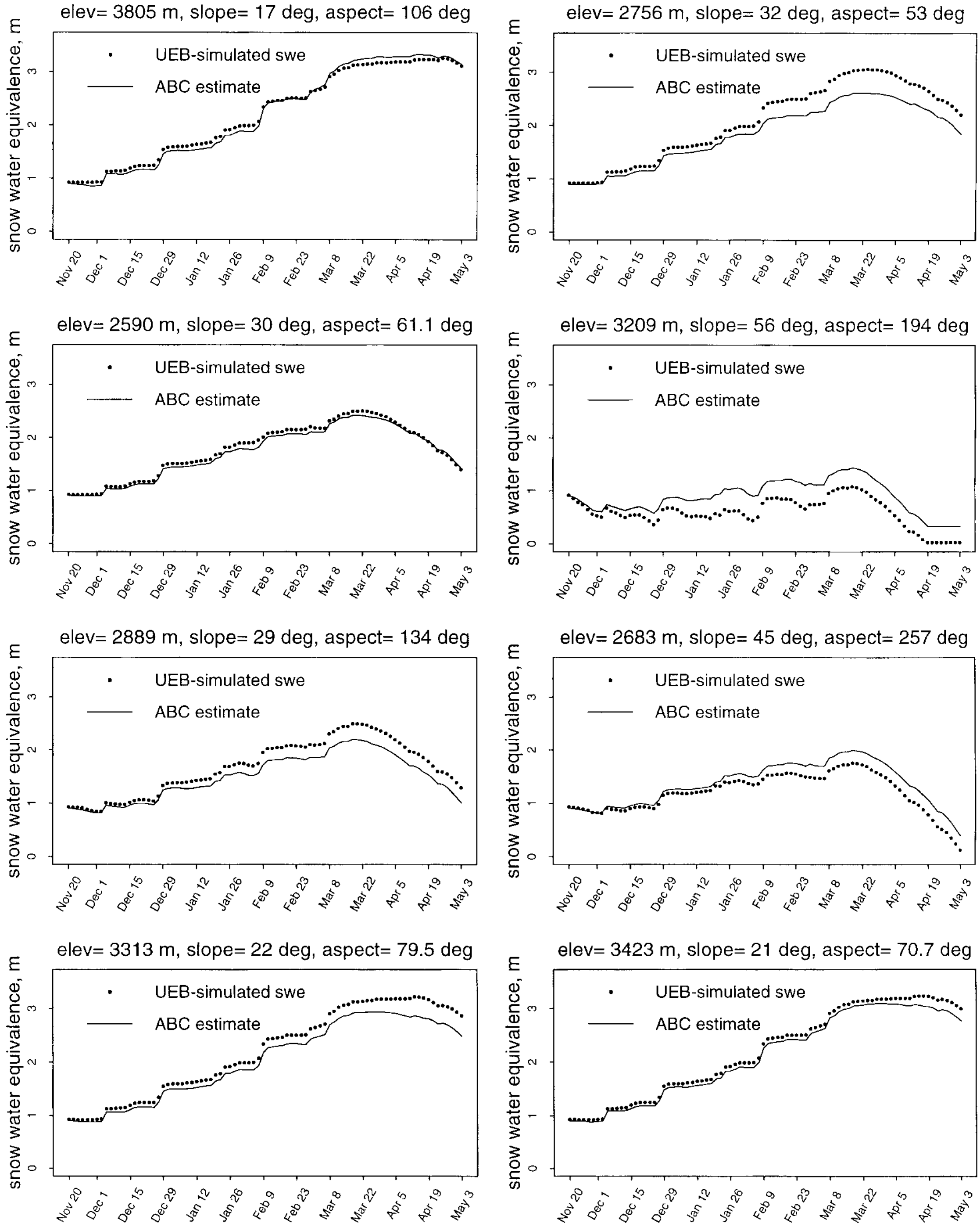

Figure 3. Snow water equivalence accumulation and melt at eight selected sites

elevation, varied in slope from 0 to 38 degrees, and faced a variety of different directions. Each point was located in the field using three wooden stakes painted white to decrease the effect of absorption and re-emission of radiation. Every four days or so the depth of snow indicated on each stake was recorded and the density of the adjacent snow was measured with a snow-tube. The SWE (snow water equivalence) was calculated as the average depth of snow recorded at the three stakes multiplied by the measured density of the snow at the site. The melt that occurred between subsequent measurements at a site was calculated by subtracting the amount of SWE measured at one time period from that of the preceding time period.

By this process, accurate measurements of the melt that occurred at each of the locations for each of the time steps were obtained (as an average of the melt occurring at the three stakes). However, small measuring errors still occurred. In a few instances, measured values of density decreased with time. Since melting snow typically becomes denser with time (unless bridging occurs, and no bridging was observed), these erroneous measurements were rectified by using average density increases instead of measured values for the sites in question. Since no new snow fell during the melt period, the one measured negative value of melt was set to zero. Williams (1998) gives full details of this data.

The melt at Smithfield Dry Canyon was modelled in the same fashion with the ABC model. Five index points were used. These points were chosen to cover a wide range of elevation and exoatmospheric radiation values.

Figure 4 shows X–Y plots of the ABC model results versus the measured melt for the 9–13 March and 13–19 March melt periods. The same index points were used in both cases, and are shown as triangles. The top graph (9–13 March) has an R^2 value of 0·47, while the lower graph has an R^2 value of 0·72. It is interesting to note the predictions for the three points represented as +'s in these figures. These data points were collected in a large, new drift that formed around 7 March. They were therefore composed of deeper, fresh snow, and the energy during the first few days contributed towards metamorphism or ripening of the snow rather than melt. This is evidenced by an under-prediction of melt in the first period, while in the second period, after the snowpack had ripened, the predictions are more in line with observed melt. If these three points are ignored in the first graph, the calculated R^2 value increases significantly to 0·60.

DISCUSSION

In evaluating the ABC model some may question the utility of a model that requires inputs of melt measurements within the basin being modelled. Traditional energy balance models require inputs of weather driving variables, measured at a point and then extended or extrapolated over a watershed. The position here is that point measurements of melt may be as easy to obtain. The purpose of the ABC model is in estimating spatially distributed melt from point snowmelt measurements. It provides a method to directly use topographic information (with digital elevation data now readily available) in a spatially distributed watershed model. Snowmelt is an inherently nonlinear process involving nonlinear relationships between forcing variables (the weather) and surface energy exchanges. However we suggest here that much of the spatial variability in snowmelt can be related linearly (to a relatively good level of approximation) to elevation and the radiation index. Thus the ABC model provides a way to separate the nonlinear variability of snowmelt at a point from the linear and predictable causes for spatial variability. As currently presented the ABC model is not useable without the point snowmelt measurements as inputs. These capture the nonlinearity in the snowmelt dynamics at a point. A relatively easy extension (left to future research) would be to couple the ABC model to a point snowmelt model run at the index points. This approach would then provide another way to obtain spatial snowmelt inputs. The difference to an explicitly spatially distributed model, is that the weather inputs are not extrapolated to each point in the watershed. Rather the weather inputs drive a point model at the index points and then the topographic information (elevation and radiation index) are used to spatially extrapolate modelled melt to each point in the watershed. Additional future work is required to fully evaluate whether predictions from an ABC like model that spatially extrapolates point

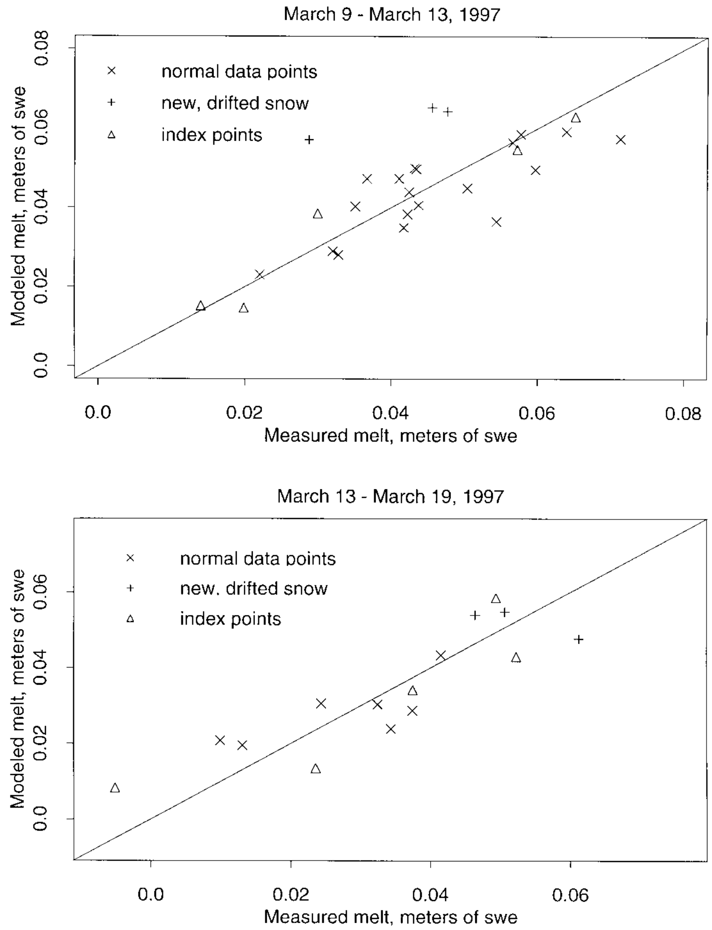

Figure 4. Comparison of ABC modelled melt against Smithfield Dry Canyon Measurements

snowmelt measurements or predictions instead of extrapolating point weather measurements is more accurate or useful.

The question of utility for forecasting and general applicability also arises with parameters A, B and C being estimated at each time step. The coefficients A, B and C should not be regarded as model parameters in the usual sense. A, B and C are not quantities that are transferable from one watershed to another, or into the future. Rather they represent the factorization of snowmelt input energy for a particular basin and time step dependent upon weather conditions into components related to radiation, elevation and a spatial constant. They can be evaluated, given topographic data, using the regression procedure, as soon as the driving inputs, snowmelt measurements at the index points, become available. They are effectively therefore internal model calculations. The model does not have any real calibration parameters. Its parameters are the topography, as quantified by the elevation and radiation index.

The results above used five index points in a very small area (~40 ha). We have not yet determined the index point density required for extension of this approach to larger areas. This is a question for future research related to the scale of variability of snow accumulation and melt and its causes. Seyfried and Wilcox (1995) discuss some of the issues involved. Out intuition is that topography (through drifting, radiation and orographic effects) accounts for a large part of the small scale variability in snow accumulation and melt up to scales of 10's to 100's of km. Beyond these scales regional weather variability becomes more important.

Since the ABC model incorporates topography, our intuition (that needs future research to verify it) is that a small number of index points (10 to 15) should work for areas up to on the order of 100 km^2.

CONCLUSIONS

The snowmelt modelling literature points to the need for a model that is both simple enough to use in practical applications for melt estimations over large areas, and rigorous enough to capture the fundamental physics of melt and to provide spatially explicit estimations. This paper has described the ABC model, which is a new method for estimating the spatial distribution of snowmelt based on point measurements and topography.

The results indicate that the ABC model can be an accurate, efficient way to predict the spatial distribution of snowmelt in a rugged, mountainous watershed. Because of the model's simplicity and accuracy, it could replace more traditional methods for modelling snowmelt in many applications. The advantages of the ABC model include:

1. simple data requirements,
2. methodology that is easily understandable to an operator,
3. computationally efficient algorithm for rapid simulations of large basins,
4. no need for prior calibration of basin-specific parameters,
5. direct incorporation of topographic information in a physically justifiable manner,
6. accurate spatial melt estimates.

Though energy balance models will always be required for some applications, there are many practical uses for the simpler ABC model. Flood forecasters as well as reservoir operators, who traditionally use simpler lumped index models, would benefit from the improved accuracy over the entire range of possible weather conditions provided by a more physically based model. Researchers could employ the model to efficiently and accurately calculate the spatial distribution of water inputs to large watersheds. This would be useful for basin response modelling, contaminant transport modelling, erosion modelling, etc. Finally, developing countries with limited finances could benefit greatly from a model that required simple data, little skill from the operator, and no prior calibration of basin-specific parameters.

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REFERENCES

Anderson EA. 1976. *A Point Energy and Mass Balance Model of a Snow Cover*. US National Oceanic and Atmospheric Administration NOAA Technical Report NWS 19, Silver Spring, MD.

Brubaker K, Rango A, Kustas W. 1996. Incorporating radiation inputs into the snowmelt runoff model. *Hydrological Processes* **10**: 1329–1343.

Cline D. 1997. Snow surface energy exchanges and snowmelt at a continental, midlatitude Alpine site. *Water Resources Research* **33**(4): 689–701.

Dingman SL. 1994. *Physical Hydrology*. Prentice-Hall: New Jersey; 575 pp.

Dingman SL, Barry RG, Reynolds RC III 1988. Application of kriging to estimating mean annual precipitation in a region of orographic influence. *Water Resources Bulletin* **24**: 329–339.

Dozier J, Frew J. 1990. Rapid calculation of terrain parameters for radiation modeling from digital elevation data. *IEEE Transactions on Geoscience and Remote Sensing* **28**(5): 963–969.

Flerchinger GN. 1987. *Simultaneous Heat and Water Model of a Snow-Residue-Soil System*. Ph.D. Dissertation, Engineering Science: Washington State University, Pullman, Washington.

Flerchinger GN, Saxton KE. 1989. Simultaneous heat and water model of a freezing snow-residual-soil system, I. Theory and development. *Transactions of the ASAE* **32**(2): 565–571.

Jordan R. 1991. *A One-Dimensional Temperature Model for a Snow Cover, Technical Documentation for SNTHERM.89*. Special Report 91-16: US Army Corps of Engineers, CRREL.

Kustas WP, Rango A, Uijlenhoet R. 1994. A simple energy budget algorithm for the snowmelt runoff model. *Water Resources Research* **30**(5): 1515–1527.

Kuusisto E. 1986. The energy balance of a melting snow cover in different environments, In *Modeling Snowmelt-Induced Processes*, EM Morris (ed.); IAHS Publication No. 155: Wallingford, UK.

Lowe PR. 1977. An approximating polynomial for the computation of saturation vapor pressure. *Journal of Applied Meteorology* **16**: 100–103.

Martinec J, Rango A, Roberts R. 1994. *The Snowmelt-Runoff Model (SRM) Users Manual*, Updated Edition 1994, Version 3.2: Department of Geography — University of Bern.

McManamon A, Day GN, Carroll TR. 1993. Snow estimation — a GIS application for water resources forecasting. *Proceedings of the Symposium on Engineering Hydrology*. ASCE Publ.: New York; p. 856–861.

Mendenhall W, Sincich T. 1992. *Statistics for Engineering and the Sciences, 3rd Edition*. Macmillan: New York p. 963.

Seyfried MS, Wilcox BP. 1995. Scale and the nature of spatial variability: field examples having implications for hydrologic modeling. *Water Resources Research* **31**(1): 173–184.

Tarboton DG. 1994. *Measurement and Modeling of Snow Energy Balance and Sublimation from Snow*. Proceedings, International Snow Science Workshop, Snowbird, Utah, October 31–November 2 1994, p. 260–279.

Tarboton DG, Chowdhury TG, Jackson TH. 1995. A spatially distributed energy balance snowmelt model. In *Biogeochemistry of Seasonally Snow-Covered Catchments*, Tonnessen KA, Williams MW, Tranter M (eds); Proceedings of a Boulder Symposium, July 3–14: IAHS Publ. no. 228: p. 141–155.

Tarboton DG, Luce CH. 1996. *Utah Energy Balance Snow Accumulation and Melt Model (UEB): Computer model technical description and user's guide*. Utah Water Research Laboratory and USDA Forest Service Intermountain Research Station.

Williams KS. 1998. *The Development and Validation of a Model for the Spatial Distribution of Snowmelt Based on Topography and Point Melt Measurements*. M.S. Thesis: Utah State University, Logan, Utah.

