CEE7430 Homework 6: Multivariate, Multisite and Disaggregation models

Due: 3/19/09

Reading:

- Loucks, D. P., J. R. Stedinger and D. A. Haith, (1981), Water Resource Systems Planning and Analysis, Prentice-Hall, Englewood Cliffs, NJ, 559 p. Chapter 6 sections 6.7 and 6.8
- Stedinger, J. R. and R. M. Vogel, (1984), "Disaggregation Procedures for Generating Serially Correlated Flow Vectors," Water Resources Research, 20(1): 47-56.

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1. Carefully read Loucks et al. 1981 Chapter 6 sections 6.7 and 6.8 that deal with multi site streamflow models. Write a 1 page abstract summary of these sections. I am looking for you to summarize the key ideas, methods and tests involved in generalizing the single site streamflow modeling methodology to multiple sites.

2. Carefully read the paper: Stedinger, J. R. and R. M. Vogel, (1984), "Disaggregation Procedures for Generating Serially Correlated Flow Vectors," Water Resources Research, 20(1): 47-56. Write a 2 page abstract summary. I am looking for you to summarize the key ideas in the paper and comment critically on assumptions, methods, results and conclusions in terms of their correctness and significance.

3. Do Loucks et al. 1981 problem 6.2

6-2. Part of New York City's municipal water supply is drawn from three parallel reservoirs in the upper Delaware River basin. The covariance matrix and lag-1 covariance matrix, as defined in equation 6.55, were estimated based on the 50-year flow record to be (in m³/s):

$$\mathbf{S}_0 = \begin{bmatrix} 20.002 & 21.436 & 6.618 \\ 21.436 & 25.141 & 6.978 \\ 6.618 & 6.978 & 2.505 \end{bmatrix} = [\mathrm{Cov}(Q_y^i, Q_y^j)]$$

$$\mathbf{S}_1 = \begin{bmatrix} 6.487 & 6.818 & 1.638 \\ 7.500 & 7.625 & 1.815 \\ 2.593 & 2.804 & 0.6753 \end{bmatrix} = [\mathrm{Cov}(Q_{y+1}^i, Q_y^j)]$$

Other statistics of the annual flows are:

Site	*Reservoir*	*Mean Flow*	*Standard Deviation*	r_1
1	Pepacton	20.05	4.472	0.3243
2	Cannonsville	23.19	5.014	0.3033
3	Neversink	7.12	1.583	0.2696

(a) Using these data, determine the values of the **A** and **B** matrices of the lag 1 model defined by equation 6.54. Assume that the flows are adequately modeled by a normal distribution. A lower triangular **B** matrix that satisfies $\mathbf{M} = \mathbf{B}\mathbf{B}^T$ may be found by equating the elements of $\mathbf{B}\mathbf{B}^T$ to those of M as follows:

$$M_{11} = b_{11}^2 \longrightarrow b_{11} = \sqrt{M_{11}}$$

$$M_{21} = b_{11}b_{21} \longrightarrow b_{21} = \frac{M_{21}}{b_{11}} = \frac{M_{21}}{\sqrt{M_{11}}}$$

$$M_{31} = b_{11}b_{31} \longrightarrow b_{31} = \frac{M_{31}}{b_{11}} = \frac{M_{31}}{\sqrt{M_{11}}}$$

$$M_{22} = b_{21}^2 + b_{22}^2 \longrightarrow b_{22} = \sqrt{M_{22} - b_{21}^2} = \sqrt{M_{22} - M_{21}^2/M_{11}}$$

and so forth for M_{23} and M_{33}. Note that $b_{ij} = 0$ for $i < j$ and M must be symmetric because $\mathbf{B}\mathbf{B}^T$ is necessarily symmetric.

(b) Determine **A** and $\mathbf{B}\mathbf{B}^T$ for the Markov model which would preserve the variances and cross covariances of the flows at all sites at the same time and the lag 1 autocovariance of flows at each site, but not necessarily the lag 1 cross covariance of flows. Calculate the lag-1 cross covariances of flows generated with your calculated **A** matrix.

(c) Assume that some model has been built to generate the total annual flow into the three reservoirs. Construct and calculate the parameters of a disaggregation model that, given the total annual inflow to all three reservoirs, will generate annual inflows into each of the reservoirs preserving the variance and cross covariances of the flows. [*Hint:* The necessary statistics of the total flows can be calculated from those of the individual flows.]

4. Do Loucks et al. 1981 problem 6.6

6-6. (a) Assume that one wanted to preserve the covariance matrices $\mathbf{S}_0$ and $\mathbf{S}_1$ of the flows at several sites $\mathbf{Z}_y$ by using the multivariate or vector ARMA(0, 1) model

$$\mathbf{Z}_{y+1} = \mathbf{A}\mathbf{V}_y - \mathbf{B}\mathbf{V}_{y-1}$$

where $\mathbf{V}_y$ contains n independent standard normal random variables. What is the relationship between the values of $\mathbf{S}_0$ and $\mathbf{S}_1$ and the matrices $\mathbf{A}$ and $\mathbf{B}$?

(b) Derive estimates of the matrices $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{C}$ of the multivariate AR(2) model

$$\mathbf{Z}_{y+1} = \mathbf{A}\mathbf{Z}_y + \mathbf{B}\mathbf{Z}_{y-1} + \mathbf{C}\mathbf{V}_y$$

using the covariance matrices $\mathbf{S}_0$, $\mathbf{S}_1$ and $\mathbf{S}_2$.