CEE7430 Homework 5: Midterm preparation

Due: 2/24/09

Reading:
- Loucks et al. (2005) sections 8.5 and 8.6 on Multivariate, multiseason and multisite models
- Loucks et al. (1981) chapter 6 (Specifically sections 6.1-6.5 and section 6.7 on multisite models. Sections 6.1-6.5 are review from a slightly different perspective.)

These are all questions from previous midterm exams. The midterm will be open book/open notes, however you will not be allowed to use a computer. Therefore I suggest you practice by doing these questions by hand.

1. For the AR(2) process shown below

$$Z_t - 1.2 Z_{t-1} + 0.6 Z_{t-2} = a_t$$

   a) Determine whether this process is stationary.

   b) If the variance of the a_t process is 1, determine the variance of the Z_t process.

   c) Calculate values for the autocorrelation coefficient ρ_k for k = 0, 1, 2, 3, 4, and 5. Plot the ACF

2. The covariance matrix and lag-1 covariance matrix of annual flows in two nearby locations are

$$\mathbf{S_0} = \begin{bmatrix} 21.5 & 15.2 \\ 15.2 & 9.3 \end{bmatrix} = Cov(Z_t^i, Z_t^j)$$

$$\mathbf{S_1} = \begin{bmatrix} 6.7 & 4.1 \\ 2.9 & 3.5 \end{bmatrix} = Cov(Z_{t+1}^i, Z_t^j)$$

   Other statistics of the annual flows are

Site	Mean flow	Standard Deviation
1	24.3	4.64
2	12.8	3.05

   a) Consider the lag-1 markov model defined by:
      $\mathbf{Z_{t+1}} = \mathbf{A} \mathbf{Z_t} + \mathbf{B} \mathbf{V_t}$
      where $\mathbf{A}$ is a diagonal matrix. Determine the values of the matrices $\mathbf{A}$ and $\mathbf{BB^T}$ so that the variance and cross covariances of the flows at all sites and the lag-1 autocovariance of the flows at each site are preserved. (You do not need to decompose $\mathbf{B}$, just report $\mathbf{BB^T}$.)

   b) Calculate the lag-1 cross covariances of the flows generated with this model and comment on differences between the observed and model lag-1 cross covariances.

   c) Consider the aggregate flow defined as
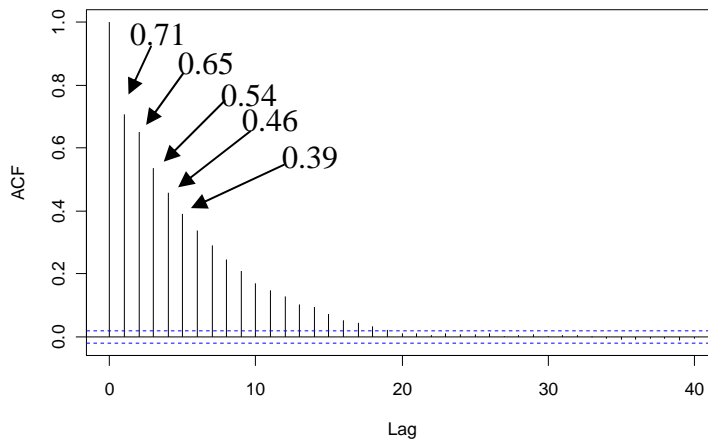 $X_t = Z_{1t} + Z_{2t}$

Determine the lag-1 correlation of the historic aggregated data, and the lag-1 correlation of the aggregated flows from the markov model developed in (a) above. Comment on any differences.

3. Given the following plots determine the type of ARMA process and estimate the parameters that you have sufficient information to estimate.
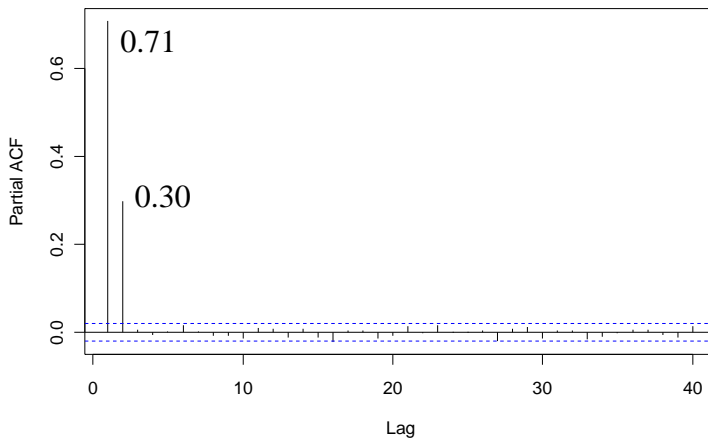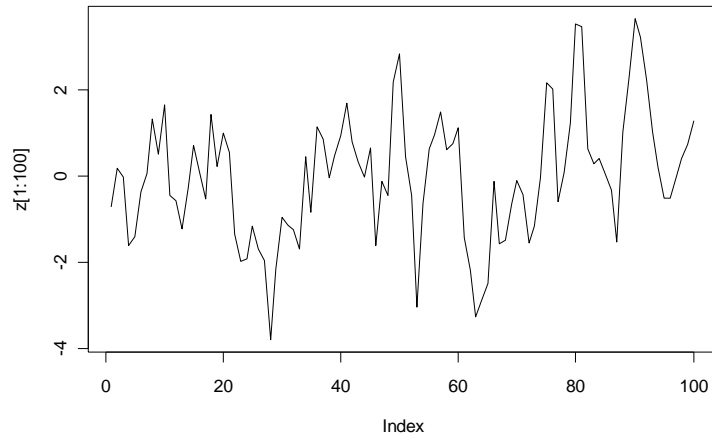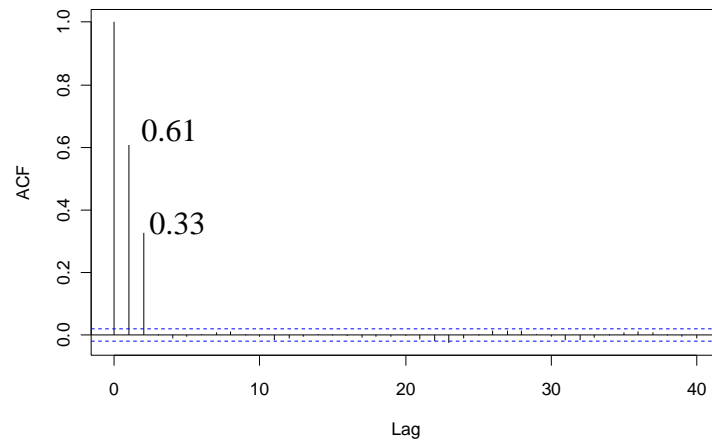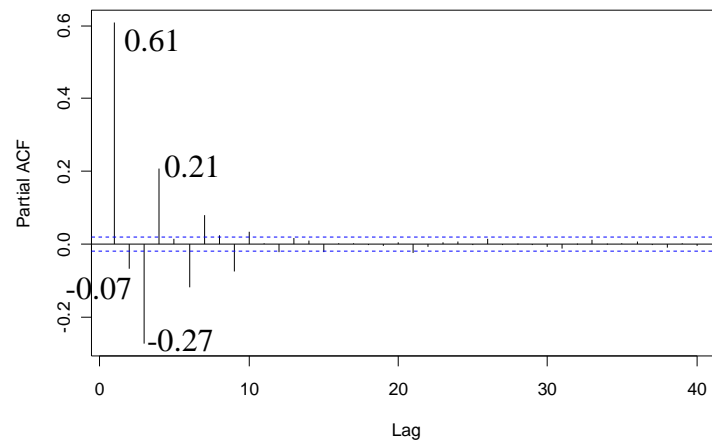
   a)
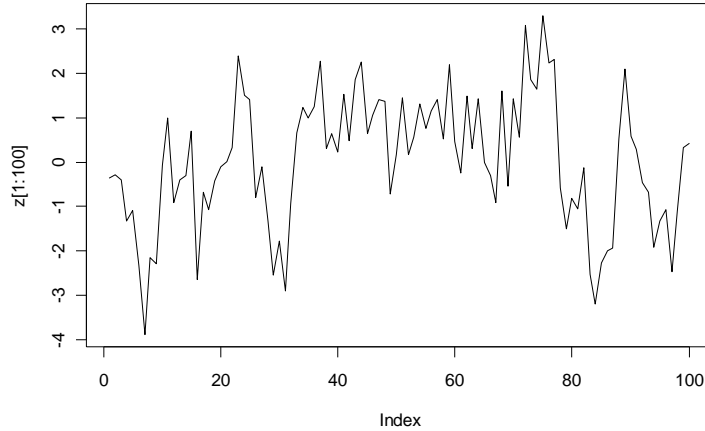
Series z

Series z

b)

Series z

Series z

4. [optional] Come up with your own suggested midterm exam questions. Provide suggestions for one or more midterm exam questions. You do not have to provide the solution. I may use questions submitted by students in the exam. Please provide these to me by 2/21/09.