Solar Radiation Direct Approach. **D.G. Tarboton**

The Dingman text uses the the equivalent slope concept for calculation of radiation (Dingman, 2002 appendix E, p 601). An alternative, and sometimes superior approach is to calculate the illumination angle (the angle between the surface normal and direction of the sun) from

$$\cos z = \cos\theta \; \cos\beta \; + \; \sin\theta \; \sin\beta \cos(A - \alpha) \qquad (1)$$

where θ is the solar zenith angle (as before from eqn E-4), β is the slope angle, α the slope azimuth (clockwise from north) and A the solar azimuth angle, the direction of the sun (clockwise from north), given by Sellers (1965 page 16)

$$\cos A = (\sin\delta - \sin\Lambda \cos\theta)/(\cos\Lambda \sin\theta) \qquad (2)$$

In this formula there is the possibility of ambiguity since $\cos^{-1}X = - \cos^{-1}X = 2\pi - \cos^{-1}X$. In the morning (wt negative) the sun is in the east and A will therefore be between 0 (north) and π (south). In the afternoon (wt positive) the sun is in the west and A will be in $(\pi, 2\pi) \equiv (-\pi, 0)$. In solving for A you should pick the solution in the appropriate range depending on the time of day. The incoming clear sky radiation at an instant on a sloping plane is then

$$I_{sc} \cos z \qquad (3)$$

or integrated over a time interval (eg. day)

$$\int I_{sc} \cos z \, dt \qquad (4)$$

This approach has the advantage that it allows one to account for shadowing, by checking whether the sun is below the local horizon in the direction A and only integrating over the time when the sun is above the horizon. Rather than integrating (3) numerically one can combine this approach with the equivalent slope concept by using (3) to solve for the instant that the sun appears or disappears due to a local horizon or obstruction, then use the exact integral of the equivalent plane approach over the appropriate time interval.

If quantities need to be evaluated per unit of horizontal area then (3) and (4) need to be divided by $\cos\beta$

Instantaneous: $I_{sc} \cos z / \cos\beta$
Daily average: $(1/\cos\beta)\int I_{sc} \cos z \, dt$

References
Dingman, S. L., (2002), Physical Hydrology, 2nd Edition, Prentice Hall, 646 p.
Sellers, W. D., (1965), Physical Climatology, University of Chicago Press, Chicago, 272 p.