CEE3430 Engineering Hydrology

Infiltration example problem

Consider a soil of given type (e.g. **silty clay loam**) and given an input rainfall hyetograph, calculate the infiltration and the runoff. Initial soil moisture content 0.3. Rainfall rate 2 cm/hr, for 3 hours.

This is an event based calculation of runoff

Solution outline

1. Determine soil properties from texture (Table 1 p 4:18)

   These are the parameters of the problem (time invariant quantities that describe behavior in a particular situation).

K_{sat}	
n	
ψ_a	
b	

2. System state described by initial condition and the depth of water that has infiltrated up to any point in time

$\theta_o = 0.3$	F = 0 cm at t=0 cm (will change during course of the event)

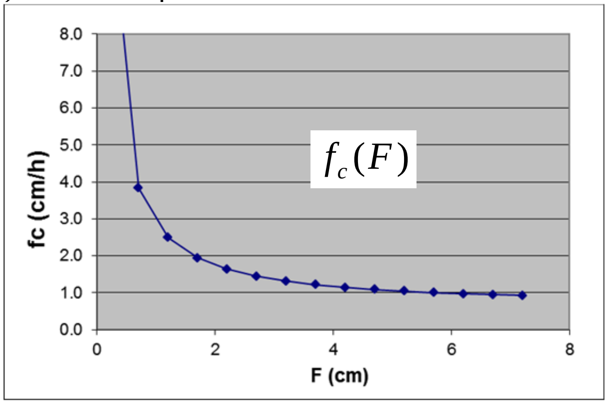
3. Establish Infiltrability – Depth approximation. In Green-Ampt approach this is based on hydraulic gradient over the depth of penetration of wetting front, Darcy's equation and suction in advance of a wetting front (Infiltration18.pptx, slide 13)

$$f_c = K_{sat}\left(1+\frac{|\psi_f|\Delta\theta}{F}\right) = K_{sat}\left(1+\frac{P}{F}\right)$$

$|\psi_f| = \frac{2b+3}{2b+6}|\psi_a|$ equation 44	
$\Delta\theta = n - \theta_o$	
P	

F cm				
f_c cm/h				

$f_c(F)$ relationship serves as foundation for calculations that follow



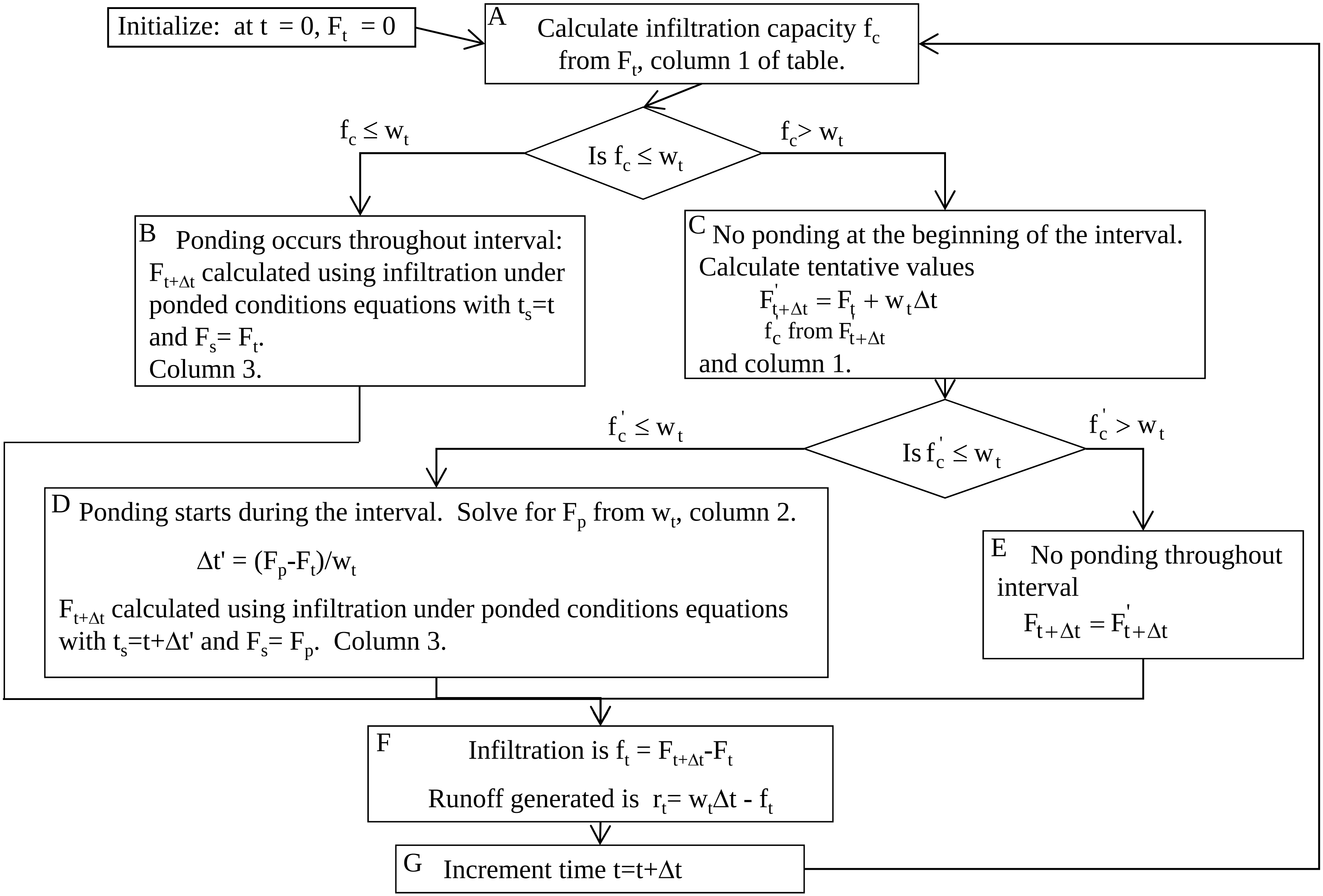
4. Ponding (saturation at the surface) first occurs when f_c=W (water input rate or rainfall rate) This idea lets you solve for the depth of water that has to infiltrate before ponding occurs, F_p, and the time to ponding t_p for a particular input rate W.

F_p cm	
t_p h	

5. After ponding the rate of increase in F (remember, this is a state variable describing the state of the system) is limited by the infiltration capacity

$$\frac{dF}{dt} = f_c(F) = K_{sat}\left(1 + \frac{P}{F}\right)$$

Solving this gives an equation relating F and t for ponded conditions


Initialize: at t = 0, $F_t = 0$
A Calculate infiltration capacity f_c from F_t, column 1 of table.
Is $f_c \le w_t$
$f_c \le w_t$
$f_c > w_t$
B Ponding occurs throughout interval: $F_{t+\Delta t}$ calculated using infiltration under ponded conditions equations with t_s=t and $F_s = F_t$. Column 3.
C No ponding at the beginning of the interval. Calculate tentative values
$F'_{t+\Delta t} = F_t + w_t \Delta t$
f'_c from $F'_{t+\Delta t}$
and column 1.
Is $f'_c \le w_t$
$f'_c \le w_t$
$f'_c > w_t$
D Ponding starts during the interval. Solve for F_p from w_t, column 2.
$\Delta t' = (F_p - F_t)/w_t$
$F_{t+\Delta t}$ calculated using infiltration under ponded conditions equations with $t_s = t + \Delta t'$ and $F_s = F_p$. Column 3.
E No ponding throughout interval
$F_{t+\Delta t} = F'_{t+\Delta t}$
F Infiltration is $f_t = F_{t+\Delta t} - F_t$
Runoff generated is $r_t = w_t \Delta t - f_t$
G Increment time $t = t + \Delta t$

Equations for variable surface water input intensity infiltration calculation.

	Infiltration capacity	Cumulative infiltration at ponding	Cumulative infiltration under ponded conditions
Green-Ampt Parameters K_{sat} and P	$f_c = K_{sat}\left(1+\frac{P}{F}\right)$	$F_p = \frac{K_{sat}P}{(w-K_{sat})}$ $w > K_{sat}$	$t - t_s = \frac{F-F_s}{K_{sat}} + \frac{P}{K_{sat}}\ln\left(\frac{F_s+P}{F+P}\right)$ Solve implicitly for F
Horton Parameters k, f_o, f_1.	$F = \frac{f_o - f_c}{k} - \frac{f_1}{k}\ln\left(\frac{f_c - f_1}{f_o - f_1}\right)$ Solve implicitly for f_c given F	$F_p = \frac{f_o - w}{k} - \frac{f_1}{k}\ln\left(\frac{w-f_1}{f_o-f_1}\right)$ $f_c < w < f_o$	Solve first for time offset t_o in $F_s = f_1(t_s - t_o) + \frac{(f_o - f_1)}{k}(1-e^{-k(t_s - t_o)})$ then $F = f_1(t - t_o) + \frac{(f_o - f_1)}{k}(1-e^{-k(t - t_o)})$
Philip Parameters K_p and S_p	$f_c\,(F) = K_p + \frac{K_p S_p}{\sqrt{S_p^2 + 4K_p F} - S_p}$	$F_p = \frac{S_p^2(w - K_p/2)}{2(w-K_p)^2}$ $w > K_p$	Solve first for time offset to in $t_o = t_s - \frac{1}{4K_p^2}\left(\sqrt{S_p^2 + 4K_p F_s} - S_p\right)^2$ then $F = S_p(t-t_o)^{1/2} + K_p(t-t_o)$

Short Example. Calculation of runoff using the Horton infiltration equation.

Parameters

f_o (cm/h) 6

f_1 (cm/h) 1

k (h^{-1}) 2

Δt (h) 0.50

Column 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
Time	Incremental Rainfall	Rainfall Intensity	F_t	$g(f_c)$	f_c	F'	$g(f_c')$	f_c'	F_p or F_s	dt'	t_s	t_o	$h(t_o)$	$F_{t+\Delta t}$	**Infiltration**	**Runoff**
(h)	(cm)	(cm/h)	(cm)		(cm/h)	(cm)		(cm/h)	(cm)	(h)	(h)	(h)		(cm)	**(cm)**	**(cm)**
0	0.7															
0.50	2															
1.00	0.8															
1.50	1															
2.00																