Snow, Snowpacks and Runoff

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Show and Tell

I've brought a tiny marvel of nature: a single snowflake.

I think we might all learn a lesson from how this utterly unique crystal turns into an ordinary molecule of water just like every other one. When you bring it in the classroom.

And now, while the analysis sinks in, I'll be leaving you...oh, okay, I'll be leaving you here. And going outside.
Potential effects of climate change on snow

- Advance in snowmelt timing
- Smaller snow pack area
- Uncertainty in regional variability

(need more research to examine trends, patterns, feedback and nonlinear dynamics. e.g. Mote, 2003, GRL)

It may be obvious to this audience, but just to set the stage, in considering snow and runoff some of the potential effects of climate change are: (read from slide). These speak to the need to examine trends and understand the interactions involved.
Quantifying snowmelt runoff requires knowledge of:

- the quantity of water held in snow packs
- the magnitude and rate of water lost to the atmosphere by sublimation
- the timing, rate, and magnitude of snow melt
- the fate of melt water

If we are to quantify snowmelt runoff we need to know the following four things: (read from slide). These involve combining observation and modeling, so a theme that I will promote is learning through integration of observation and modeling.
Attributes of Snow Important in Runoff

- Snow Water Equivalent
- Heterogeneity
- Thermal properties

In understanding snow and runoff it is important to quantify these things. Snow water equivalent is the depth of water that results when a snowpack melts and it is the basic quantity used to quantify the amount of snow water present. We do not have a good way to measure or predict snow water equivalent over a large watershed due in large part to spatial heterogeneity. We can measure SWE at a point, but over an area it is difficult. I'll talk about some of the methods in a minute, but measuring SWE patterns over large areas is a significant limitation that needs attention. Snow is also a very interesting substance. Being comprised of ice crystals and air it acts as an insulator to the conduction of heat. Heat transport processes also modify the medium itself. These properties need to be understood and reflected in models to correctly estimate snow melt surface water inputs. I'll talk about some of the recent ideas and improvements in modeling snow thermal properties.
This reviews some of the methods for quantifying snow water equivalent. Ground based methods include field snow surveys and the SNOTEL system based on snow pillows. These have the disadvantage of being point measurements that are hard to extend over a large watershed. Remote sensing offers the only real opportunity to obtain spatial snow water equivalent and here are some of the methods. (Discuss advantages and limitations). I think that to make real progress in quantifying SWE we really need to integrate models with measurements, an assimilation approach. NOHRSC has made quite a bit of progress in this area, but more is needed.
Now I would like to really focus on heterogeneity of snow at multiple scales. Heterogeneity is caused by a number of processes - listed here. **Quantifying and understanding heterogeneity is a major challenge for snow science.**
Hydrologic Importance of Heterogeneity

- Sustains surface water input late into season
- Limits exposure of snow surface to sublimation

Upper Sheep Creek, Photograph Keith Cooley
Approaches to spatially distributed modeling of snow over a catchment

- Effective or representative parameters.
- Variability parameterized using distribution functions.
- Combination of representative points (e.g. fuzzy membership).
- Indexed semi-distributed (e.g. elevation zones or HRU's).
- Detailed spatially explicit (e.g. grid cells).
I now go through a series of examples to illustrate the state of the art and future challenges. This slide shows the difference between applying a point model and detailed spatially explicit model over a small area to estimate surface water input. The spatially distributed model sustains surface water input from the drift areas much later in the spring, more consistent with the observed timing of snowmelt runoff. The spatially distributed model used the accumulation factor concept to relate measured precipitation to accumulation at each location accounting for redistribution due to drifting. In this example at Upper Sheep Creek, accumulation factor was estimated from measurements of SWE at each grid point.
Measurements of SWE are hard to obtain over a larger area so other approaches are needed to estimate accumulation factor. This work from Jinsheng You uses a method based on snow covered area to estimate accumulation factor. The idea is that at each point there is a time/date at which snowcover was last observed and a later date at which a snow free surface was first observed. The accumulation factor in the mass balance equation can be used to adjust a model to have it predicting snow disappearance on either of these dates. This gives the range of AF. Now if we look at edge grid cells in the images and make the assumption that if they are snow covered, they are about to become snow free, or if they are snow free that they just became snow free then one of the bounds on AF can be selected for a subset of the domain. This subset of edge cells is used to calibrate a regression model related to topographic attributed to estimate AF over the entire domain.
Another approach to modeling the spatial variability of SWE is to use a blowing snow model to physically quantify the effect of snow drifting. This slide illustrates the Snowtran 3D model developed by Liston and Sturm that represents the processes listed.
Here is an example of application of this model over the upper portion of the Reynolds Creek experimental watershed. Point comparisons were disappointing, but the overall distribution of SWE compared not too badly between model and observations. In some applications it is only this distribution that is needed so these results are still useful. They also point to the need for improving blowing snow models.
Accumulation Factor estimated from topography using empirical rules

\[(a_1 + a_2z)(1-f(slope))(1+a_3\text{curv})\]


This slide illustrates an empirical approach based on topography developed by Günter Blöschl.
This is also a slide from Blöschl's work. In snow modeling the scale of model elements controls the extent to which spatial variability can be explicitly represented by the model. Here in this spectral density function the variability associated with each scale or frequency is depicted. On the left at spacings larger than the element size variability is explicitly represented by differences between model elements. On the right at spacings smaller than the model element size variability has to be represented within a model element, i.e. subgrid variability. The representation of subgrid variability has been the focus of much work in snow and other modeling.
Depletion curve for parameterization of subgrid variability in accumulation and melt

\[ A_f(W) = A_f^* \left( \frac{W}{W_{\text{max}}} \right) \]

\[ A_f(m) = \int_{0}^{\infty} f_g(w + m)dw \]
\[ = \int_{0}^{\infty} f_g(w)dw = 1 - F_g(m) \]


This slide shows a method we developed to use a depletion curve to represent subgrid variability.
Depletion curve parameterization of a point snowmelt model

\[ W_{sc} = \frac{W_a}{A_f \left( \frac{W_a}{W_{a\ max}} \right)} \]

\[ \frac{dU}{dt} = Q_{sn} + Q_{li} - Q_{le} + Q_e + Q_h + Q_g + Q_p - Q_m \]

\[ \frac{dW_{sc}}{dt} = P_f + P_s - M_r - E \]

This slide shows how a depletion curve is used to evolve a point model using energy and mass fluxes over only the snow covered area.
This shows how this works, in comparison to observations and a spatially explicit fully distributed model.
In the slide just presented the depletion curve was derived based upon detailed spatially distributed measurements. This is impractical for large areas so we need ways to relate the depletion curves to surrogate measures of variability. This slide shows depletion curves derived from the following surrogate variables. (1) Elevation, (2) accumulation factor, in this case estimated from the snow covered area images, and (3) a peak accumulation estimate based on regression with accumulation factor and elevation as predictor variables. The regression parameters were calibrated against peak accumulation from a spatially explicit model. The spatially explicit distributed model is used as a reference. We see that it is possible to get depletion curves that approach the reference depletion curve based on surrogate variables, but that the better estimates still require some reliance on spatially explicit modeling.
In this slide we examine the time stability of depletion curves. One of the foundations of the depletion curve approach is that the same shape of depletion curve is assumed for low and high snow years. The rescaling based on maximum accumulation is important for the ease of practical implementation free from specific date assumptions for peak accumulation. This shows in data from Keith Cooley at Upper sheep creek for 9 years that the depletion curve (at least here) appears to be relatively stable across years. This stability is the basis for another idea for the use of depletion curves. **Snow covered area** is relatively easy to measure, and if one during a melt season observed $A_f$, one can infer from a depletion curve the fraction of basin average snow water equivalent that has melted and the fraction that remains in the basin. One then just needs an index of scale to infer the actual melt amounts or water equivalent remaining. Potential indices include point measurements, e.g. from SNOTEL, or measurements of melt runoff at a stream gage. This idea is developed further in a minute.
Practical estimation of depletion curves remains a challenge

A suggestion
Assimilate observed snow covered area with modeling to derive depletion curve and surface water input

There are still challenges in the estimation of depletion curves and following is a suggestion (not tested) that we put forth for possibly doing this. Again this relies on the idea that snow covered area is relatively easy to measure. One can take measurements of snow covered area over time to establish $A_f(t)$. 
Point model initialized with sufficient snow to sustain snowcover to season end

Next one can initialize a model with sufficient snow to sustain snow cover over a season to obtain potential melt over time.
Combining these gives $A_f(m)$. The relationship between snow covered area and potential melt is at the heart of the derivation of depletion curves from the spatial distribution of snow as illustrated here. Given an initial spatially variable snow distribution subject to potential melt $m$, the remaining snow water equivalent is the integral of snow water equivalent over the area with depths greater than $m$. This establishes the relationship between basin average snow covered area $W_a$ and melt $m$. This can be used to establish the depletion curve $W_a(A_f)$ or its inverse.
Potential results

- Post season reconstruction of $W_a(t)$

- $A_f(W_a)$ for future modeling and surface water input estimation

- Once depletion curve is in hand observations of $A_f$ give fraction of $W$ that has been depleted/is remaining

This idea has potential to provide for post season reconstruction of time series of basin average snow water equivalent. Don Cline and colleagues used a similar idea to reconstruct spatially distributed snow water equivalent based on a spatially distributed model. This is also similar to the idea Jinsheng You's results that I showed earlier for estimating accumulation factor from a sequence of images. In these methods I think that uncertainty will be reduced when the function is well defined by frequent snow covered area images. MODIS may be good sensor to try with this method. Once the depletion curve is established through this approach it can be used for future modeling and surface water estimation, as well as in real time to give the fraction of snow remaining in a watershed for operational purposes.
The role that snow plays in energy fluxes also deserves some attention. This shows thermocouple measurements of snow temperature at different depths in a snowpack as well as infrared measurements of surface temperature. The dampening of fluctuations with depth illustrate the thermal insulating effect of snow.
Heat conduction theory

Semi infinite domain
\[ \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial z^2} \quad k = \frac{\lambda}{C \rho} \]

For diurnal fluctuations at surface
\[ T(z, t) = T_0 + A e^{\frac{z}{2k}} \sin \left( \frac{\omega_1 t - \frac{z}{d_i}}{d_i} \right) \]
\[ \omega_1 = \frac{2\pi}{24 \text{ hr}} \quad d_i = \sqrt{\frac{2k}{\omega_1}} \]
\[ Q_{\text{av}} = -\lambda \left. \frac{dT}{dz} \right|_{z=0} = \frac{\lambda}{d_i \omega_1} \frac{dT_s}{dt} + \frac{\lambda}{d_i} (T_s - <T>) \]

These observations can be understood using heat conduction theory over a semi-infinite domain. For diurnal fluctuations at the surface the dampening depth is dependent upon snow properties. This idea was used in the UEB snowmelt model we developed to parameterize the surface temperature without having to resort to layering in the model. These observations can also be used to infer thermal properties of the snow for use in models.
Inference of thermal properties

Amplitude decrease used to evaluate effective conductivity to each depth

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<th>Amplitude (C)</th>
<th>exp(z/d₁)</th>
<th>z/d₁ (cm)</th>
<th>d₁ (cm)</th>
<th>k (cm²/hr)</th>
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This slide shows some snow properties estimated from these measurements.
Snow temperature measurements useful to track and validate model energy content needed to predict for the onset of snowmelt

This slide shows comparisons of modeled and measured snow energy content. We found these measurements of snow temperature useful to validate modeled energy content and correct energy content modeling discrepancies.

Important issues not mentioned

- Sublimation
- Interception and the role of vegetation
- Local advection effects on energy balance
- Within snow flow processes resulting in
  - Fingers
  - Ice columns and lenses
  - Implications for runoff generation of concentrated surface water input

I have in this presentation focused on the importance of measuring, understanding and modeling heterogeneity, and to a lesser extent touched on some of the thermal properties of snow. There are also many things that I did not mention because of time limitations that are also important and are listed here.
Challenges to understanding snow and runoff processes in a variable climate

- Quantifying and measuring input precipitation/snowfall
  - basin average and spatial patterns
- Measuring SWE over large areas
  - spatial patterns
- Integration of remote sensing and ground observations and modeling

Let me try wrap up some of these ideas. Here is a list of some of the challenges I see where additional snow hydrology research is needed.
Challenges contd.

- Quantifying and understanding heterogeneity
  - Improve blowing snow models
  - Improve relationships with topography (generalization)
  - Improve representations of sub grid variability (watershed scale and up)
- Better quantify snow thermal properties