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HYDROLOGIC SAMPLING — A CHARACTERIZATION IN TERMS OF RAINFALL AND BASIN PROPERTIES

RAFAEL L. BRAS, DAVID G. TARBOTON and CARLOS PUENTE*

Department of Civil Engineering, Massachusetts Institute of Technology, Cambridge, MA 02139 (U.S.A.)

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ABSTRACT

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This paper considers the sampling of rainfall and discharge processes both in time and in space and links the sampling problem to basin and rainfall characteristics. The effectiveness of different sampling strategies is measured by the variance of the error in estimating either total or peak of streamflow from a single storm event. This is related to the rainfall and basin rainfall–discharge properties through parameterizations of these processes. Rainfall is modeled as a collection of rain cells which occur randomly in space and time and has parameters which define the probability of occurrence of rain cells in space and time and the spread of rainfall due to a cell. Discharge from rainfall is parameterized in terms of the fluvial geomorphology of the basin. Linear filtering techniques are used to compute the variance of the estimation error for different sampling strategies. Sampling strategies are defined by the number of rain gages, rainfall sampling interval and discharge measurement interval. The results can be used in hydrologic network design to assess the effectiveness of different sampling options.

NOTATION

A	Area	$[L^2]$
$a(t)$	Random process time varying parameter	
$a_2(t)$	Random process parameter for $p_2(t)$	
α	Rain cell decay parameter	$[T^{-1}]$
B^2	Subscript to represent D^2 or $D^2 + \sigma^2$ in $C_{B^2}(d)$	$[L^2]$
β	Cell birth parameter (temporal)	$[T^{-1}]$
Cov ( )	Covariance operator	
$C_{B^2}(d)$	Spatial component of rainfall covariance function	
$C(A, B^2)$	Area variance reduction factor	
D	Rain cell spread parameter	[L]
d	Distance, usually between two points z_1, z_2	[L]
Δt	Generic sampling interval, may refer to rainfall or flow	[T]
Δt_p	Rainfall sampling interval	[T]
Δt_q	Flow sampling interval	[T]
$E(\,)$	Expectation, i.e., mean value operator	

*Present address: Department of Land, Air and Water Resources, University of California, Davis, CA 95616 (U.S.A.)

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$e(t)$	Rainfall sampling error process	[L/T]
$e_2(t)$	Sampling error process of $\zeta_2(t)$ (cluster process)	[L/T]
ε, ε_2	Velocity correction parameters	[T^{-1}]
$\boldsymbol{F}$	State transition matrix	
$F(\ )$	Function defined in eqn. (48)	
$f_1(\ ), f_2(\ )$	Generic functions	
$q_{i_0}(t, r)$	Rainfall intensity from a cell	[L/T]
$\Gamma(\ )$	Gamma function	
$\gamma(t_1, t_2)$	Autocovariance function for point rainfall	
$\gamma_2(t_1, t_2)$	Autocovariance function of clustering process	
$h(t)$	Cumulative discharge	
θ	Rainfall sampling error to process variance ratio	
θ_2	Sampling error to process variance ratio for cluster process	
$I(t)$	Instantaneous unit hydrograph	
i	Summation index, or index of location	
i_0	Rain cell center rainfall rate (i.e., intensity)	[L/T]
k	River basin scale parameter	[T]
L	Length of highest order stream	[L]
$l(t)$	Cumulative observable rainfall	[L]
λ	Cell birth parameter (spatial)	[L^{-2}]
m	River basin shape parameter/number of Nash reservoirs	
$m(t)$	Mean rainfall intensity	[L/T]
N	Number of rain gages	
ν	Number of rain cells in a cluster	
$p(t)$	Area averaged rainfall rate	[L/T]
$\hat{p}(t)$	Observable rainfall rate	[L/T]
$p_2(t)$	Area averaged $\zeta_2(t)$ (cluster process)	[L/T]
π	Number pi $\simeq 3.14159\ldots$	
$Q(t)$	River basin discharge rate	[L/T]
$Q_1, Q_2 \ldots Q_m$	Discharge from Nash reservoirs	[L/T]
$q(t)$	White noise spectral density (i.e., variance)	
$q_2(t)$	Spectral density of white noise $w_2(t)$	
$q_2^{\varepsilon}(t)$	Spectral density of white noise $w_2^{\varepsilon}(t)$	
R_A	Horton area ratio	
R_B	Horton bifurcation ratio	
R_L	Horton length ratio	
R^2	Two-dimensional space	
r	Radial distance from cell center	[L]
ρ	Cluster spatial distribution parameter	[L^{-2}]
σ^2	Variance of distance of cell birth from cluster center	[L^2]
σ_T^2	Variance in total rainfall/discharge	[L^2]
t	Time, when subscripted denotes different times, with convention $t_2 \geqslant t_1$	[T]
t_m	Time of peak mean flow rate	[T]
t_L	Forecast lead time	
τ	Time of birth of cell	[T]
$\boldsymbol{u}$	Cell velocity vector	[L/T]
v	Flow velocity in streams	[L/T]
$w(t)$	White noise	
$w^{e}(t)$	White noise of sampling error process	
$w_2(t)$	White noise of process $p_2(t)$	
$w_2^{\varepsilon}(t)$	White noise of process $\varepsilon_2(t)$	
$\boldsymbol{w}$	White noise vector	
ω	Stream order index	

ω_i	Weight factor in estimation of area rainfall	
$x(t)$	Generic random process	
$\boldsymbol{x}$	State vector	
$\chi(d\tau, dy)$	Cell births counting process	
y	Spatial location (coordinate vector) of cell center	[L]
z	Spatial location (coordinate vector), when subscripted denotes different locations	[L]
$\zeta(t, z)$	Point rainfall rate (i.e., intensity)	[L/T]
$\zeta_1(t, z)$, $\zeta_2(t, z)$	Artificial processes which when added give point rainfall rate. $\zeta_2(t)$ is called the cluster process	[L/T]

INTRODUCTION

This paper describes a study of the sampling of rainfall and discharge in an interrelated fashion. Since discharge is directly due to rainfall, it is the contention here that rainfall and discharge sampling network design should be considered together. Measures of the effectiveness of combined rainfall and flow measurements are developed in terms of the variance of sampling error. Tarboton et al. (1987) addressed this problem by considering rainfall to have a separable, stationary covariance structure. The concept is extended here to include nonstationary models of the rainfall process. Rainfall is modeled as a collection of rain cells which occur randomly in space and time. The formulations of Waymire et al. (1984) and Rodriguez-Iturbe and Eagleson (1987) are used.

Linear systems theory is used to link the precipitation (input) and discharge (output). This implies assumptions of linearity in the basin response, which can be represented by a convolution equation. Discharge is parameterized in terms of the fluvial geomorphology of the basin. The next two sections give details of these parameterizations. A sampling strategy is defined as the number of rain gages within a basin and the intervals between rain and flow measurements. A state space approach is used to formulate the minimum variance linear estimate of streamflow due to a storm, given a particular strategy for rain and flow measurements. Since linear theory is being used, the variance of this estimate can be computed before any measurements are made. This variance is used to define the effectiveness of the sampling strategy and can be used to compare different sampling strategies when designing hydrologic sampling networks.

PARAMETERIZATION OF RAINFALL

The structure of rainfall patterns has been studied extensively using radar and detailed rain gage measurements. Austin and Houze (1972) and Harrold (1973) identify structures which consist of rain cells embedded within mesoscale precipitation areas within synoptic storm systems. Waymire et al. (1984) give a model which conceptually incorporates this hierarchical structure. Rodriguez-Iturbe and Eagleson (1987) develop models along similar lines and derive moments of the multidimensional probability distribution of occurrences. These results are in the form of nonstationary expected values and covariance functions for rainfall intensity. These functions are used here to parameterize the rainfall process.

Two models for the distribution of rain cells are used. The first assumes that rain cells are distributed uniformly (according to a Poisson distribution) in space. The second assumes that rain cells occur around cluster points, according to a Neyman–Scott type clustering process.

Model with Poisson occurrence of rain cells in space

It is assumed that each storm is made up of a random number of rain cells which occur in space and time according to a three-dimensional point process. Each cell spreads its rain according to a function $g_{i_0}(t - \tau;\ |z - y|)$ which represents the rainfall intensity at time t from the beginning of the storm on point z from a cell born at time τ at point y. The function $g_{i_0}(\cdot)$ is taken to have the structure:

$$g_{i_0}(t, r) = \begin{cases} i_0 \mathrm{e}^{-\alpha t} \mathrm{e}^{-r^2/2D^2}, & \text{for } t \geqslant 0 \\ 0 & \text{for } t < 0 \end{cases} \tag{1}$$

where i_0, the cell center intensity, is an independent random variable, α and D are parameters defining the temporal and spatial extent of the rain cell. The rainfall intensity at any point is represented by:

$$\zeta(t, z) = \int_{R^2} \int_{-\infty}^{\infty} g_{i_0}[(t - \tau), |z - y|)]\ \chi(\mathrm{d}\tau, \mathrm{d}y) \tag{2}$$

where $\chi(\mathrm{d}\tau, \mathrm{d}y)$ represents the number of cells born in the infinitesimal region $\mathrm{d}\tau$, $\mathrm{d}y$ near (τ, y). Integration is over the two-dimensional space R^2 and all time, although with the definition of g_{i_0} in eqn. (1) the integrand is only nonzero for $\tau < t$, i.e., cells initiated before time t. It is assumed that the spatial and temporal occurrences of cells are independent events and also that cells occur spatially according to a two-dimensional Poisson process with parameter $\lambda\,[\mathrm{L}^{-2}]$ and temporally (from the beginning of the storm) according to an exponential distribution with parameter $\beta\,[\mathrm{T}^{-1}]$. With these assumptions Rodriguez-Iturbe and Eagleson (1987) derive the nonstationary mean and covariance functions of the process. For a storm with rain cells moving with velocity $\boldsymbol{u}$. Tarboton (1987) obtains the following results which involve an approximation to the covariance expression:

$$E[\zeta(t, z)] = \frac{E(i_0)\lambda\beta 2\pi D^2}{\alpha - \beta} (\mathrm{e}^{-\beta t} - e^{-\alpha t}) = m(t) \tag{3}$$

$$\mathrm{Cov}[\zeta(t_1, z_1), \zeta(t_2, z_2)] = \frac{\lambda\beta E(i_0^2)\pi D^2}{2\alpha - \beta} \mathrm{e}^{-(\alpha+\varepsilon)t_1} [\mathrm{e}^{(\alpha+\varepsilon-\beta)t_2} - \mathrm{e}^{-(\alpha-\varepsilon)t_2}]\mathrm{e}^{-d^2/4D^2} \tag{4}$$

where here d is the distance between points z_1 and z_2 and $\varepsilon = |\boldsymbol{u}|/2D$. To simplify notation we have used the convention $t_1 \geqslant t_2$.

This covariance function is of the separable form:

$$\mathrm{Cov}[\zeta(t_1, z_1), \zeta(t_2, z_2)] = \gamma(t_1, t_2) C_{D^2}(|z_1 - z_2|) \tag{5}$$

where $C_{D^2}(d) = \mathrm{e}^{-d^2/4D^2}$ and $\gamma(t_1, t_2)$ is a nonstationary autocovariance function for point rainfall.

Model with cells clustered in space

Rain cells are now assumed to be distributed in space according to a two-dimensional Neyman–Scott process. Cluster centers occur according to a Poisson process with parameter ρ. These cluster centers are not rain cells, but are just points around which the density of cells is larger than in other regions. Each cluster has associated with it a number of rain cells v, which is a random variable, independent and identically distributed for each cluster center. The birth points (τ, y) of cells follow distributions that are exponential in time with parameter α and multivariate normal in space, centered (mean) at the cluster center x with variance σ^2. The spread of rain from each cell is the same as in non-clustered case, given by eqn. (1). With these assumptions Rodriguez-Iturbe and Eagleson (1987) derive the nonstationary mean and covariance functions of the process. Again, approximating their results to account for moving cells in the covariance function, Tarboton (1987) gives:

$$E[\zeta(t, z)] = \frac{E(i_0)\rho\beta E(v) 2\pi D^2}{\alpha - \beta}(\mathrm{e}^{-\beta t} - \mathrm{e}^{-\alpha t}) = m(t) \tag{6}$$

$$\begin{aligned}\mathrm{Cov}[\zeta(t_1, z_1), \zeta(t_2, z_2)] = {} & \frac{\rho E(v)\beta E(i_0^2)}{2\alpha - \beta} \mathrm{e}^{-(\alpha-\varepsilon)t_1} [\mathrm{e}^{(\alpha+\varepsilon-\beta)t_2} - \mathrm{e}^{-(\alpha-\varepsilon)t_2}] \pi D^2 \mathrm{e}^{-d^2/4D^2} \\ & + \frac{E[v(v-1)]\rho\beta^2 E^2(i_0)\pi D^4}{(\alpha-\beta)^2(D^2+\sigma^2)}(\mathrm{e}^{-\beta t_1} - \mathrm{e}^{-\alpha t_1}) \\ & \times (\mathrm{e}^{-\beta t_2} - \mathrm{e}^{-\alpha t_2}) \mathrm{e}^{\varepsilon_2 t_1} \mathrm{e}^{-\varepsilon_2 t_2} \exp\left[-\frac{d^2}{4(D^2+\sigma^2)}\right]\end{aligned} \tag{7}$$

where here d is the distance between points z_1 and z_2 and $\varepsilon_2 = |\boldsymbol{u}|/2\sqrt{D^2 + \sigma^2}$. Again the convention is $t_1 \geqslant t_2$. Notice that with λ replaced by $\rho E(v)$, eqn. (6) is identical to the nonclustered mean, eqn. (3) and that the first term of eqn. (7) is identical to eqn. (4). The second term of eqn. (7) can be expressed in terms of the mean, leading to:

$$\mathrm{Cov}[\zeta(t_1, z_1), \zeta(t_2, z_2)] = \gamma(t_1, t_2)\, C_{D^2}(|z_1 - z_2|) + \gamma_2(t_1, t_2)\, C_{D^2+\sigma^2}(|z_1 - z_1|) \tag{8}$$

where $\gamma(t_1, t_2)$ is analogous to the Poisson case with $\lambda = \rho E(v)$:

$$\gamma_2(t_1, t_2) = \frac{E[\nu(\nu - 1)]}{E^2[\nu]4\pi\rho(D^2 + \sigma^2)} m(t_1)m(t_2)\, e^{-\varepsilon_2(t_1 - t_2)} \tag{9}$$

and:

$$C_{B^2}(d) = e^{-d^2/4B^2} \tag{10}$$

where B^2 can represent D^2 or $D^2 + \sigma^2$. For the clustered case it is convenient to think of $\zeta(t, z)$ as the sum of two independent processes $\zeta_1(t, z)$ and $\zeta_2(t, z)$, i.e.:

$$\zeta(t, z) = \zeta_1(t, z) + \zeta_2(t, z) \tag{11}$$

with $\zeta_1(t, z)$ having all the properties of $\zeta(t, z)$ in the unclustered case and $\zeta_2(t, z)$ having 0 mean and covariance:

$$\mathrm{Cov}[\zeta_2(t_1, z_1), \zeta_2(t_2, z_2)] = \gamma_2(t_1, t_2)\, C_{D^2+\sigma^2}(|z_1 - z_2|) \tag{12}$$

Area averaged rainfall

Our interest here is in the area average precipitation intensity since this is the input to a lumped rainfall–runoff model. This is defined:

$$p(t) = \frac{1}{A} \int_A \zeta(t, z)\, dz \tag{13}$$

Clearly from (3) and (6), $E[p(t)] = m(t)$, since the mean is independent of spatial location.

The covariance is:

$$\mathrm{Cov}[p(t_1), p(t_2)] = \gamma(t_1, t_2) \frac{1}{A^2} \int_A \int_A C_{B^2}(|z_1 - z_2|)\, dz_1\, dz_2 \tag{14}$$

For the clustered rainfall model this spatial averaging is applied separately to $\zeta_1(\cdot)$ and $\zeta_2(\cdot)$ in (11). Define:

$$C(A, B^2) = \frac{1}{A^2} \int_A \int_A C_{B^2}(|z_1 - z_2|)\, dz_1\, dz_2 \tag{15}$$

This is an area reduction factor since it relates the point covariance to the area averaged autocovariance. For rectangular regions this integral has been evaluated and is given in Fig. 1.

In practice it is not possible to measure the area averaged process, eqn. (13). It is approximated from point rain gage measurements as follows:

$$\hat{p}(t) = \sum_{i=1}^{N} \omega_i\, \zeta(t, z_i) \tag{16}$$

where ω_i is a set of weights corresponding to N gages at locations z_i. Other investigators (Rodriguez-Iturbe and Mejia, 1974; Lenton and Rodriguez-Iturbe, 1974; Bras and Rodriguez-Iturbe, 1975) have done work on the positioning of the

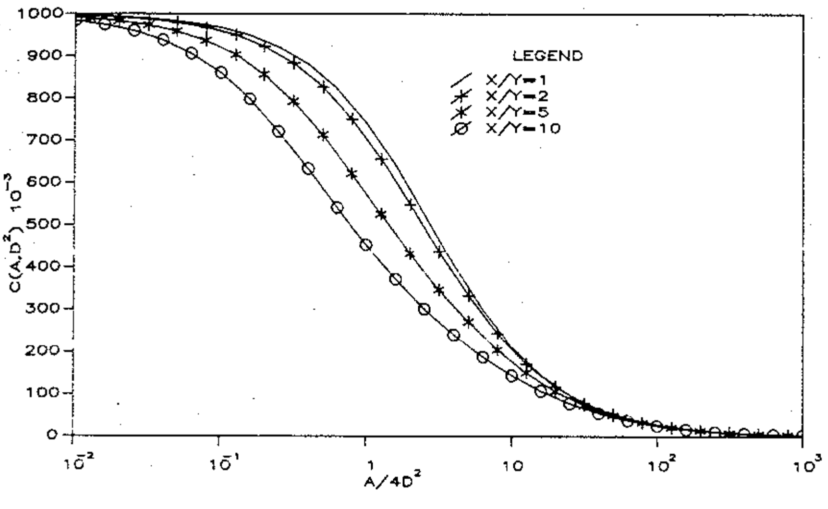

Fig. 1. Area reduction factor for a rectangular region with a double exponential correlation.

gages and the choice of weights to optimize this approximation. From eqns. (16) and (13) we can define the sampling error process:

$$e(t) \quad = \quad \hat{p}(t) - p(t) \tag{17}$$

Then using eqns. (5), (8) and (13)–(15) we can obtain:

$$\mathrm{Cov}[e(t_1), e(t_2)] \quad = \quad \gamma(t_1, t_2)\left[\sum_{i=1}^{N}\sum_{j=1}^{N} \omega_i C_{B^2}(|z_i - z_j|) - 2\sum_{i=1}^{N}\frac{\omega_i}{A}\int_A C_{B^2}(|z_i - z|)\,\mathrm{d}z + C(A, B^2)\right] \tag{18}$$

and:

$$\mathrm{Cov}[e(t_1), p(t_2)] \quad = \quad \gamma(t_1, t_2)\left[\sum_{i=1}^{N}\frac{\omega_i}{A}\int_A C_{B^2}(|z_i - z|)\mathrm{d}z - C(A, B^2)\right] \tag{19}$$

In principle these can be evaluated for a given set of gage locations z_i. The optimal gage locations z_i and weights w_i could be obtained by minimizing (18) for a given N. If locations are fixed the optimal weights are obtained by invoking a principle of orthogonality between process and errors and setting (19) equal to zero. We choose to avoid these issues, so for simplicity we assume $\omega_i = 1/N$ and take the gages as randomly located. We then get taking expectations over the random location of gages in (18) and (19):

$$\mathrm{Cov}[e(t_1), e(t_2)] \quad = \quad \gamma(t_1, t_2)\,\frac{1 - C(A, B^2)}{N} \tag{20}$$

and:

$$\mathrm{Cov}[e(t_1), p(t_2)] = 0 \tag{21}$$

Other methods of gage location will be analogous except for a different multiplicative constant in (20). Equation (21) will hold for any optimal set of weights, i.e., process and error are orthogonal.

BASIN RESPONSE

The basin response is parameterized in terms of its instantaneous unit hydrograph. This gives runoff via the convolution:

$$Q(t) = \int_0^t I(\tau)\, p(t - \tau) \mathrm{d}\tau \tag{22}$$

where $p(t)$ is the area averaged rainfall intensity, $I(t)$ the basin's transfer function or instantaneous unit hydrograph (IUH), and $Q(t)$ the runoff at time t.

Here we use an approximation to Rodriguez-Iturbe and Valdes (1979) geomorphologic instantaneous unit hydrograph (GIUH) suggested by Rosso (1984). Rosso uses a two parameter gamma probability density function to represent the IUH:

$$I(t) = [k\Gamma(m)]^{-1}(t/k)^{m-1}\exp(-t/k) \tag{23}$$

where k = scale parameter; m = shape parameter; and $\Gamma()$ = gamma function.

The parameters m and k are related to geomorphology by:

$$m = 3.29(R_B/R_A)^{0.78} R_L^{0.07} \tag{24}$$

$$k = 0.70[R_A/(R_B R_L)]^{0.48}\left(\frac{L}{v}\right) \tag{25}$$

where R_B, R_A, R_L are the Horton numbers: *bifurcation* ratio, *area* ratio, and *length* ratio, respectively; L is the length of the highest order stream; and v is the peak velocity on the streams, assumed constant over the basin. The Horton numbers R_B, R_A and R_L are the constants in Horton's well-known empirical laws and are obtained from the channel network ordered according to Strahler's ordering scheme. Quantitatively, these are $R_B = N_\omega/N_{\omega+1}$, $R_L = L_\omega/L_{\omega-1}$ and $R_A = A_\omega/A_{\omega-1}$ where N_ω is the number of streams of order ω, L_ω is the mean length of streams of order ω and A_ω is the mean area of basins of order ω. For further details see Strahler (1964). Equation (23) can be conceptualized as a cascade of linear reservoirs with storage constant k (the Nash (1957) model). Basin response can be described by a finite number of differential equations:

$$\left.\begin{aligned}
\frac{\mathrm{d}Q_1}{\mathrm{d}t} &= -\frac{1}{k}Q_1 + \frac{1}{k} \times \text{inflow} \\
\frac{\mathrm{d}Q_2}{\mathrm{d}t} &= \frac{1}{k}Q_1 - \frac{1}{k}Q_2 \\
&\;\;\vdots \\
\frac{\mathrm{d}Q_m}{\mathrm{d}t} &= \frac{1}{k}Q_{m-1} - \frac{1}{k}Q_m
\end{aligned}\right| \qquad (26)$$

where Q_i is flow from the ith linear reservoir.

Here inflow is taken as the area averaged (lumped) rainfall with no consideration for infiltration. In future work the effects of infiltration and its variability will be considered.

STATE SPACE FORMULATION

In a previous section the covariance functions of area average rainfall intensity and the difference between this and average rainfall intensity at the gages was developed. This section uses these functions to formulate a linear stochastic model of the rainfall process and measurement system. These are augmented with the differential equations describing the basin response to obtain a combined state space model of the rainfall runoff process. First some general results are given that help us obtain a stochastic model from a covariance function.

The stochastic model used here will be a first-order nonstationary stochastic differential equation of the form:

$$\frac{\mathrm{d}}{\mathrm{d}t}x(t) = a(t)x(t) + w(t) \qquad (27)$$

where $x(t)$ is a zero mean process and $w(t)$ is white noise with nonstationary spectral density $q(t)$. The processes we wish to model have covariance functions in time of the form:

$$\mathrm{Cov}(t_1, t_2) = f_1[\max(t_1, t_2)]\, f_2[\min(t_1, t_2)] \qquad (28)$$

Note that $\gamma(t_1, t_2)$ and $\gamma_2(t_1, t_2)$, the functions we are interested in, are of this form. It can be shown that (Tarboton, 1987) $a(t)$ and $q(t)$ are related to the parts of the covariance function $f_1(\cdot)$ and $f_2(\cdot)$ by:

$$a(t) = f_1(t)/f_1(t) \qquad (29)$$

and:

$$q(t) = f_1(t)f_2'(t) - f_2(t)f_1'(t) \tag{30}$$

Here the prime notation is used to denote the derivative. Equation (30) places a restriction on the forms of (28) that can be modeled by (27) since $q(t)$ must be positive. This condition is satisfied in our case.

Using eqns. (29) and (30) with the Poisson rainfall covariance function, eqn. (14) leads to:

$$a(t) = -(\varepsilon + \alpha) \tag{31}$$

and:

$$q(t) = C(A, D^2)\lambda\beta E(i_0^2)\pi D^2\left[e^{-\beta t} + \frac{2\varepsilon}{2\alpha - \beta}(e^{-\beta t} - e^{-2\alpha t})\right] \tag{32}$$

A differential equation describing the area integrated rainfall process deviation from mean is therefore:

$$\frac{d}{dt}[p(t) - m(t)] = -(\varepsilon + \alpha)[p(t) - m(t)] + w(t) \tag{33}$$

where $w(t)$ is white noise with time varying spectral density $q(t)$.

Similarly applying (29) and (30) to the sampling error covariance equation (20) results in:

$$\frac{d}{dt}e(t) = -(\varepsilon + \alpha)e(t) + w^e(t) \tag{34}$$

where $w^e(t)$ is white noise with time varying variance $[1 - C(A, D^2)]/[N\,C(A, D^2)]\,[q(t)]$. Note that the factor $\theta = [1 - C(A, D^2)]/[N\,C(A, D^2)]$ gives the ratio of rainfall sampling error to process variance, and is an important variable in assessing the performance of the rainfall sampling network.

For the clustered rainfall model we treat the two parts $\zeta_1(t)$ and $\zeta_2(t)$ separately, see eqn. (11). $\zeta_1(t)$ is completely analogous to the Poisson rainfall, so can be modeled by eqns. (33) and (34). Applying eqns. (29) and (30) to $p_2(t)$, the area averaged form of $\zeta_2(t)$ gives:

$$a_2(t) = \frac{\alpha e^{-\alpha t} - \beta e^{-\beta t}}{e^{-\beta t} - e^{-\alpha t}} - \varepsilon_2 \tag{35}$$

and:

$$q_2(t) = 2C(A, D^2 + \sigma^2)\varepsilon_2[m(t)]^2\frac{E[\nu(\nu - 1)]}{E^2[\nu]4\pi\rho(D^2 + \sigma^2)} \tag{36}$$

The stochastic differential equation describing $p_2(t)$ is therefore:

$$\frac{d}{dt}p_2(t) = a_2(t)\,p_2(t) + w_2(t) \tag{37}$$

where $w_2(t)$ is white noise with time varying spectral density $q_2(t)$.

The error process $e_2(t)$ can be modeled by an analogous process with white noise $w_2^e(t)$ that has covariance given by $q_2^e(t) = [1 - C(A,\ D^2 + \sigma^2)]/[N\ C(A,\ D^2 + \sigma^2)][q_2(t)]$. Here the factor $\theta_2 = [1 - C(A,\ D^2 + \sigma^2)]/[N\ C(A,\ D^2 + \sigma^2)]$ gives the ratio of error to process variance for the cluster process.

Rainfall observations are in fact generally cumulative over the observation interval. To allow for these it is convenient to define:

$$l(t) = \int_0^t [p(t) + e(t)]\mathrm{d}t \tag{38}$$

for the Poisson case and:

$$l(t) = \int_0^t [p(t) + p_2(t) + e(t) + e_2(t)]\mathrm{d}t \tag{39}$$

for the clustered case. Then an observation can be represented as $l(t) - l(t - \Delta t)$. The differential equivalents of (38) and (39) are:

$$\frac{\mathrm{d}}{\mathrm{d}t} l(t) = e(t) + p(t) \tag{40}$$

$$\frac{\mathrm{d}}{\mathrm{d}t} l(t) = e(t) + p(t) + e_2(t) + p_2(t) \tag{41}$$

A quantity of interest is often the total flow from a storm. This is mathematically:

$$h(t) = \int_0^t Q_m(\tau)\mathrm{d}\tau \tag{42}$$

Differentiating we get:

$$\frac{\mathrm{d}h(t)}{\mathrm{d}t} = Q_m(t) \tag{43}$$

The state space model of combined rainfall and runoff is formed by combining eqns. (26), (33), (34), (37), (40), (41) and (43) into a single vector stochastic differential equation in which the input is taken as the true area averaged precipitation $p(t)$, for the Poisson case or $p(t) + p_2(t)$ for the clustered case.

This is:

$$\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{x} = \boldsymbol{F}\,\boldsymbol{x} + \boldsymbol{w} \tag{44}$$

For the Poisson rainfall model, the state vector $\boldsymbol{x}$ is:

$$\boldsymbol{x} = [Q_1(t)\ \ldots\ Q_m(t),\ p(t) - m(t),\ e(t),\ l(t),\ h(t)]$$

and $\boldsymbol{w}$ is a white noise vector with a spectral density matrix with all elements zero except those on the diagonal in positions $m + 1$ and $m + 2$ which contain the spectral densities of white noise for $p(t) - m(t)$ and $e(t)$ given by eqns. (33) and (34).

For the clustered rainfall model the state vector is:

$$x = [Q_1(t) \ldots Q_m(t), p_1(t) - m(t), e_1(t), p_2(t), e_2(t), l(t), h(t)] \tag{45}$$

w for this model has a spectral density matrix with all elements zero except diagonal elements in the $m + 1$ to $m + 4$ positions corresponding to the spectral densities of white noise for $p_1(t) - m(t), e_1(t), p_2(t)$ and $e_2(t)$ given with eqns. (33), (34) and (37).

Standard linear filtering techniques are used to find linear minimum variance estimates of the states given a priori estimates at time 0 and observations from that time. The variance of the estimation error can be computed in advance. This advance computation of error variance, before any sampling is done, is used as a measure of the effectiveness of the sampling. The procedure for propagation and updating of state estimates was first presented by Kalman and is clearly explained by Brown (1983). Rainfall observations of the form $l(t) - l(t - \Delta t)$ representing measurement of accumulated rainfall require use of a delayed state filter (Brown, 1983, p. 315). Here the possibility of flow measurements occurring in the interval $(t - \Delta t, t)$ slightly complicates matters requiring extensions to the method described by Brown (1983). Tarboton (1987) gives full details of these extensions, which are technical and not necessary for understanding the remainder of this paper.

Generally, linear filtering techniques account for errors in measurement as well as uncertainty due to variability of the process and scarcity of sampling. In this study measurement errors were taken as zero. This was done for reasons of simplicity and generality. We did not want to mix the effect of high uncertainty due to poor measurements with high uncertainty due to sampling scarcity. Measurement errors are also device and scale dependent so it is difficult to obtain general results valid for many devices and scales. This study therefore focuses on the effectiveness of error free yet scarce sampling in reducing estimation error.

SAMPLING STRATEGY DESIGN

The model developed above is used as a tool to compare different sampling options. Sampling consists of rainfall measurement and flow measurement. The number of rain gages and frequency with which rainfall and flow are measured constitute a sampling strategy. Specifically a strategy is defined by the triplet $(N, \Delta t_p, \Delta t_q)$ where N is the number of rain gages; Δt_p is the rainfall measurement interval; and Δt_q is the discharge measurement interval. Prior to a storm the rainfall intensity is zero and the volume and rate of runoff from the storm are zero. These are known with certainty, so initial conditions of zero uncertainty, i.e. error covariance matrix set equal to zero, were used. One term in this matrix corresponding to the nugget variance in the clustering process had to be initialised different from zero. Tarboton (1987) gives the details of this. During a storm our knowledge of rainfall and flow rates and volumes becomes uncertain, dependent on our sampling strategy. We will use two design criteria. First the network design will be based on the asymptotic variance of cumulative flow estimates and then the criterion will be the accuracy of the prediction of peak flow for a given forecast lead time.

Variance of total flow

When considering the asymptotic variance of total flow volume it is important to realize that the basin parameterization plays no role in the process variance. This is because the unit hydrograph only controls the time at which precipitation inputs reach the output and not the question of whether they get there which has been neglected here through our neglect of infiltration. The variance of total volume can be obtained by evaluating:

$$\sigma_V^2 = E\left(\left\{\int_0^{\infty}\left[p(t) - m(t)\right]dt\right\}^2\right)$$

where $p(t)$ is given by (13), $m(t)$ is given by (6) and autocovariance is given by (14). Doing this we obtain:

$$\sigma_V^2 = \frac{\lambda E(i_0^2)\pi D^2}{\alpha(\alpha + \varepsilon)} C(A, D^2) \tag{46}$$

in the case of the Poisson rainfall model and:

$$\sigma_V^2 = \frac{\rho E(\nu)E(i_0^2)\pi D^2}{\alpha(\alpha + \varepsilon)} C(A, D^2) + \frac{E[\nu(\nu - 1)]\pi D^2 \rho [E(i_0/\alpha)]^2}{1 + \left(\dfrac{\sigma}{D}\right)^2} F\left(\frac{\beta}{\alpha}, \frac{\varepsilon_2}{\alpha}\right) C(A, D^2 + \sigma^2) \tag{47}$$

where:

$$F\left(\frac{\beta}{\alpha}, \frac{\varepsilon_2}{\alpha}\right) = \frac{(\beta/\alpha)^2}{1 - (\beta/\alpha)^2}\left[\frac{1}{\dfrac{\beta}{\alpha}\left(\dfrac{\varepsilon_2}{\alpha} + \dfrac{\beta}{\alpha}\right)} - \frac{1}{\dfrac{\varepsilon_2}{\alpha} + 1}\right] \tag{48}$$

in the case of the clustered rainfall model. Note that these are process variances, independent of the sampling strategy. To compare sampling strategies the variance of the total volume estimation error given a particular sampling strategy was normalized by dividing by values computed from (46) or (47). This gives a performance measure between 0 and 1. A possible design criterion may be the specification of acceptable normalized error variances, for example 0.01 or 0.25, corresponding to estimation error standard deviations of 10 and 50% of the process standard deviation.

At this point it is convenient to make the parameters dimensionless by normalizing over time with the cell decay parameter α and over length by the cell size parameter D. The Poisson rainfall model can then be characterized by the dimensionless parameter groupings λD^2, (β/α), (ε/α) and $E[(i_0/\alpha)^2]/[E(i_0/\alpha)]^2$.

TABLE 1

Dimensionless groups — Typical values

Group	Value	Source	Used here
λD^2	0.15	Arizona FEW	0.1
	0.0767	Assumed* RIE	
$\rho E[\nu]D^2$	0.0767	Assumed* WGR	0.1
	0.15–0.55	Assumed* VRIG	
β/α	0.264	Assumed* WGR	0.25
	0.23–0.45	Assumed* VRIG	
$\frac{\varepsilon}{\alpha} = \frac{u}{2D\alpha}$	10	Assumed* WGR	0.5
	0.06–0.19	Assumed* VRIG	
$\frac{E[(i_0/\alpha)^2]}{(E[i_0/\alpha])^2}$	1	Assumed* WGR, VRIG, corresponds to cells with constant intensity	2
	2	Assumed* RIE, FEW, corresponds to cells with exponentially distributed center intensity	
$\frac{E[\nu(\nu-1)]}{E[\nu]}$	4	Assumed* RIE	4
	10–15	Assumed* VRIG Note that ν is often assumed Poisson distributed in which case this parameter is equal to $E[\nu]$	
$1 + \left(\frac{\sigma}{D}\right)^2$	7.27	Assumed* RIE	7
	5–10	Assumed* VRIG	

*Assumed means the parameter value given was used as a typical value, without fitting to data. FEW = Fennessey et al. (1986); RIE = Rodriguez-Iturbe and Eagleson (1987); WGR = Waymire et al. (1984); and VRIG = Valdes et al. (1985).

The basin is characterized by m and αk, while the sampling strategy is characterized by $\alpha\Delta t_p$, $\alpha\Delta t_q$ and the rainfall sampling error to process variance ratio $\theta = [1 - C(A,D^2)]/[N\ C(A,\ D^2)]$. The cluster rainfall model has the above dimensionless groups, with $\rho E(\nu)D^2 \equiv \lambda D^2$, and the following additional groups:

$$\frac{E[\nu(\nu-1)]}{E(\nu)},\quad 1 + \left(\frac{\sigma}{D}\right)^2$$

$\theta_2 = [1 - C(A,D^2 + \sigma^2)]/[NC(A, D^2 + \sigma^2)]$ is required in addition to the above parameters to parameterize the sampling strategy.

There have been very few studies which have estimated parameters for the sort of rainfall model used here. Table 1 gives values that have been reported

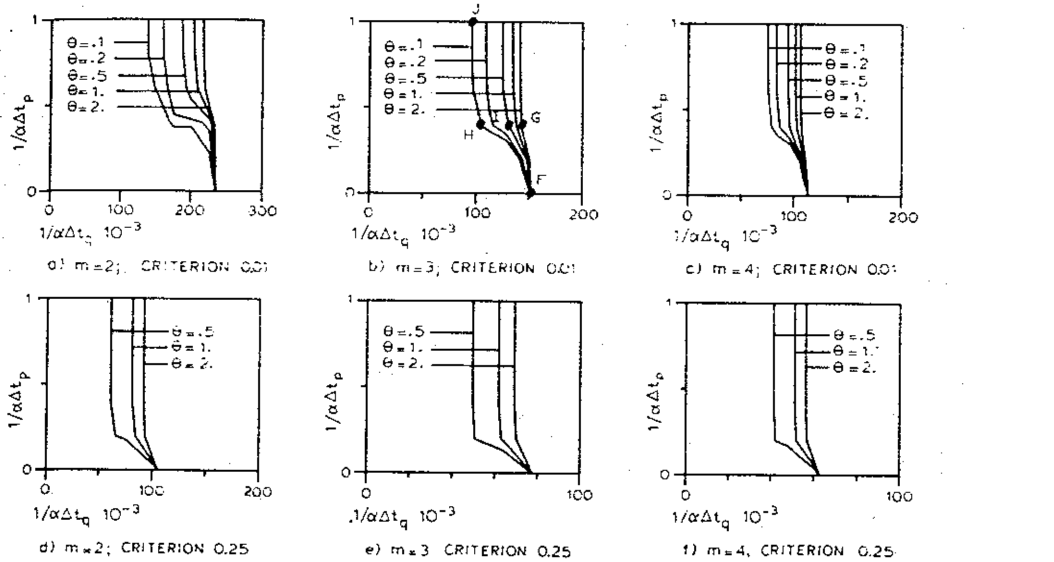

Fig. 2. Poisson rainfall model. Sampling strategy selection according to total flow variance criterion with $\alpha k = 2$.

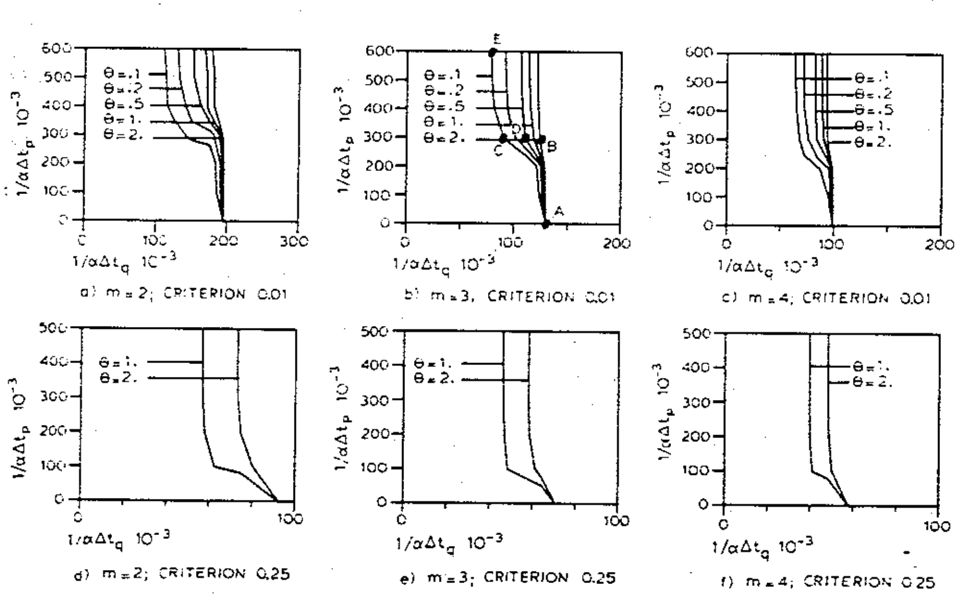

Fig. 3. Clustered rainfall model. Sampling strategy selection according to total flow variance criterion with $\alpha k = 2$.

or used in the literature, together with representative values chosen for use here as a basis for sampling strategy design.

Tarboton (1987) shows that with these parameter values the approximation

$\theta_2 = 0.15\,\theta$ is good for a wide range of basins, so for both models a sampling strategy is characterized by $\alpha\Delta t_p$, $\alpha\Delta t_q$ and θ. Figures 2 and 3 show the sampling strategies which provide nominal acceptable levels of normalized error variance of 0.01 or 0.25 in the estimation of flow volume for different combinations of basin parameters m and $\alpha k = 2$. Figure 2 corresponds to the poisson rainfall model, and Fig. 3 to the clustered rainfall model. These sampling strategy selection figures were obtained as follows. For a large number of sampling strategies (θ, $\alpha\Delta t_p$, $\alpha\Delta t_q$) the variance of estimation error was computed by numerically propagating the error variance part of the filter described in the state space formulation above. Each result was normalized by dividing by results from eqns. (46) and (47). The lines on the figures were then obtained by drawing contours of the appropriate error variances through the arrays of results obtained.

In developing these results the climate parameters at the right of Table 1 have been used. A set of figures like this for a range of parameter αk would provide a quasigeneral design aid for the selection of sampling strategies. They are quasigeneral since they can be used over a range of basin parameters, but only with the specific climate parameters from Table 1 unless other results are obtained.

Variance in prediction of peak flow

Unlike the total flow volume, the peak flow rate and variance are basin dependent. Of interest to use is the variance of the peak flow rate. This cannot be determined in a simple way since the time of the flow peak is random. An approximation is obtained by considering the variance of the flow rate at the time of the peak mean flow rate (denote this time t_m). Convoluting the basin response function, eqn. (23), with the mean rainfall rate, eqns. (3) or (6), we can find the dependence of t_m on basin parameters, Fig. 4. The dependence of the variance of flow rate at t_m on basin parameters is given in Fig. 5. Note that the variance on the vertical axis of Fig. 5 has been normalized by the area reduction factor $C(A, D^2)$. In Fig. 5b, the clustered case a large area approximation $C(A, D^2 + \sigma^2) = [1 + (\sigma/D)^2]\, C(A, D^2)$ given by Tarboton (1987) has also been used. The performance of a sampling strategy in predicting peak flow rate can be measured by comparison of the prediction error variance at t_m with the process variance at this time. In other words, Fig. 5 provides an upper bound on the error variance we can expect at t_m.

As well as being dependent on the sampling strategy, the variance in our estimate of peak flow is dependent on the forecast lead time. Denote the forecast lead time t_L, then the time into the storm from which predictions must be made is $t_m - t_L$. A lower bound on how good we can possibly expect our predictions to be is obtained by assuming perfect knowledge of all states at this time. Propagation of variance to t_m then gives the variance of predictions at t_m, based on perfect knowledge at $t_m - t_L$. Figures 6 and 7 show the dependence of variance at t_m assuming perfect knowledge at $t_m - t_L$ for different basin pa-

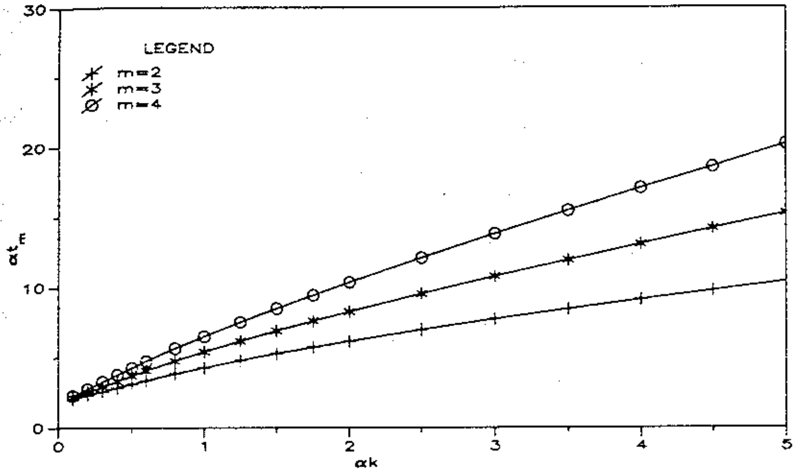

Fig. 4. Dependence of time to peak on basin parameters.

rameterizations. In these the time into the storm has been normalized by the time to peak. On the horizontal axes are the ratio of time into storm from which prediction is made, to the time to peak in the mean $[(t_m - t_L)/t_m]$. The vertical axes are error variance of the flow at t_m, divided by the process variance at t_m (given by Fig. 5). An interesting feature is that with the cluster model, Fig. 7, the normalized variance is not 1 for zero lead time. This is because the clusters result in a nugget component of variance, realized at the beginning of the storm, when cluster location was determined. Tarboton (1987) gives details of how this was accounted for, as an initial condition, when obtaining the results.

Now the design procedure may be as follows. Given basin parameters αk and m, Fig. 4 gives the time of peak mean flow. This is regarded as the critical time for design purposes, and our strategies are selected to meet acceptable levels

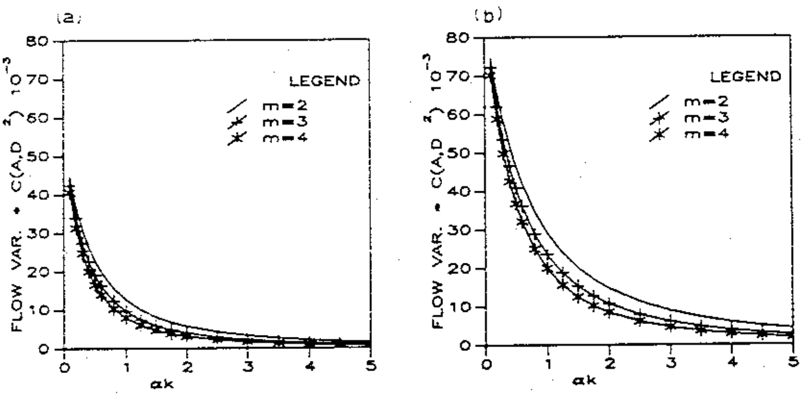

Fig. 5. Dependence of variance in flow rate at time of peak on basin parameters. (a) Poisson rainfall model; (b) clustered rainfall model.

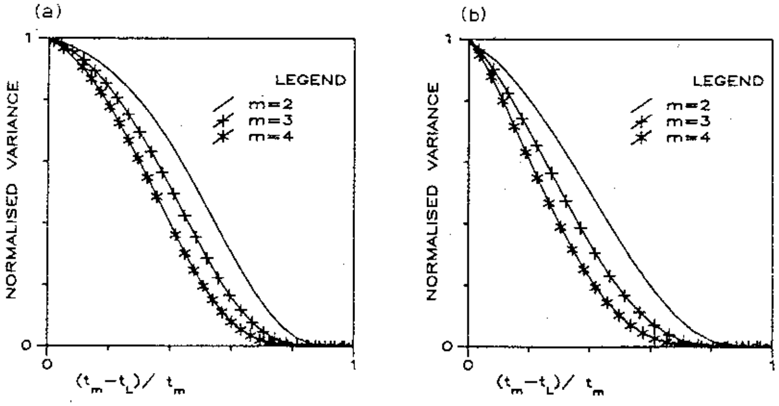

Fig. 6. Poisson rainfall model. Variance of predicted flow rate at t_m given perfect knowledge at $t_m - t_L$. (a) $\alpha k = 1$; and (b) $\alpha k = 2$.

of error at this time. Figure 5 shows the variance of flow rate at this time. The best we can possibly do, predicting from time $t_m - t_L$ is shown in Figs. 6 and 7. In practice it may be reasonable to assume some acceptable level of error above this and find sampling strategies that meet this criterion. Here we have obtained detailed results for $t_m - t_L = 2/3\, t_m$; i.e., we want to predict the peak from 2/3 of the way to the peak. This results in lead time $t_L = 1/3\, t_m$. Figures 6 and 7 show that at this lead time the best achievable normalized error variances range from 0 to 0.3. We therefore suggest that a reasonable design criterion may be normalized variance $\leqslant 0.5$. Note that the last available measurements upon which prediction is based do not necessarily occur at time $t_m - t_L$, but may occur anywhere (randomly) in the interval $(t_m - t_L - \Delta t, t_m - t_L)$, where Δt is the measurement interval. This was accounted for here by making the last measurement upon which prediction is based occur at several

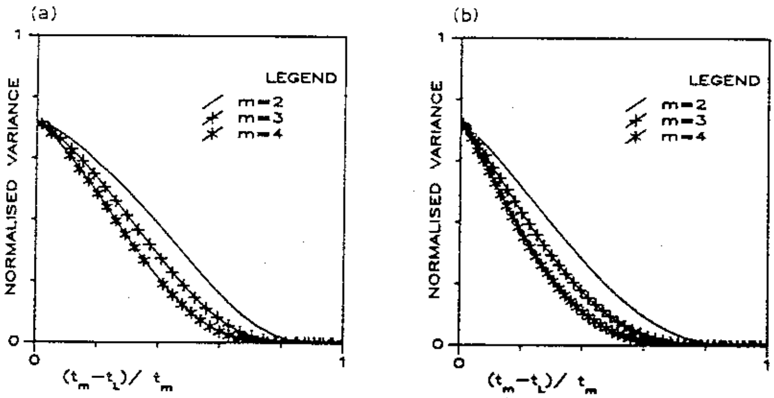

Fig. 7. Clustered rainfall model. Variance of predicted flow at t_m given perfect knowledge at $t_m - t_L$. (a) $\alpha k = 1$; and (b) $\alpha k = 2$.

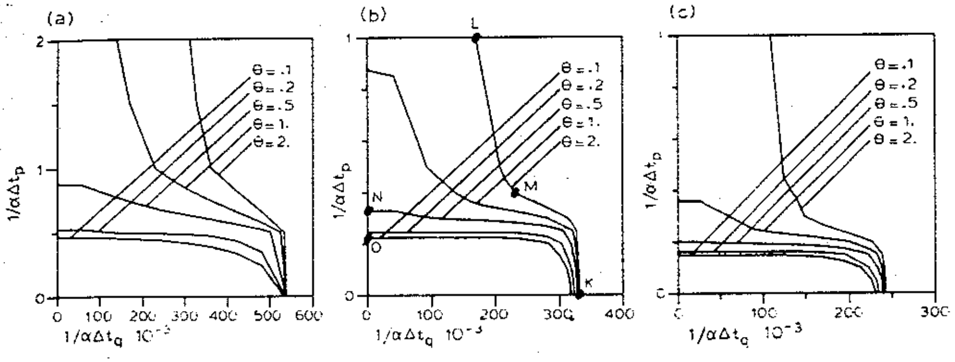

Fig. 8. Poisson rainfall model. Sampling strategy selection according to peak flow variance criterion with $\alpha k = 2$. (a) $m = 2$; (b) $m = 3$; and (c) $m = 4$.

points spaced uniformly over this interval and using the average of the variances predicted for each of these cases to compare with our criterion. Figures 8 and 9 give sampling strategies that meet this criterion, for basin parameter $\alpha k = 2$. Again this type of figure provides a quasigeneral design aid for the selection of sampling strategies, when prediction of the peak flow is important.

Example

Use of Figs. 2, 3, 8 and 9 is illustrated in the following example. A hypothetical river basin has Horton numbers $R_B = 3.5$, $R_A = 4$, $R_L = 2.5$, length of highest order stream $L = 7$ km and stream peak flow velocity is estimated to be $v = 4\,\mathrm{km\,h^{-1}}$. This basin can be approximated by a rectangle 15 km × 10 km and is situated in a region where rain cells have properties $D = 1.5$ km and $\alpha = 2\,\mathrm{h^{-1}}$. Using (24) and (25) we compute $m = 3.16$ and $k = 0.84$ h therefore $\alpha k = 1.7$.

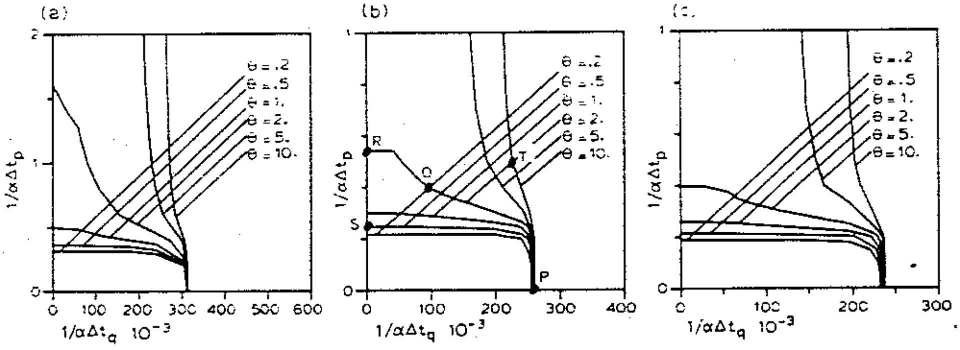

Fig. 9. Clustered rainfall model. Sampling strategy selection according to peak flow variance criterion with $\alpha k = 2$. (a) $m = 2$; (b) $m = 3$; and (c) $m = 4$.

TABLE 2

Example of selection of sampling strategy for total discharge error variance using the clustered rainfall model

Point	θ	Rainfall sampling frequency $1/(\alpha\Delta t_p)$	Discharge sampling frequency $1/(\alpha\Delta t_q)$	Rainfall sampling interval (h) Δt_p	Discharge sampling interval (h) Δt_q	Number of rain gages N
A	–	0	0.13	∞, none	3.8	0
B	2	0.3	0.125	1.67	4	$2.8 \simeq 3$
C	0.1	0.3	0.09	1.67	5.6	57
D	0.5	0.3	0.115	1.67	4.3	11
E	0.1	0.6	0.08	0.83	0.25	57

The nearest approximation to these in a set of design aid figures may be Figs. 2, 3, 8 and 9 corresponding to $m = 3$, $\alpha k = 2$. Suppose we wish the normalized variance of our total flow estimates not to exceed 0.01; i.e., Figs. 2b and 3b apply. Strategies for points marked A through E on Fig. 3b and F through J on 2b are given in Tables 2 and 3. The value $C(A,D^2) = 0.15$ obtained from Fig. 1 has been used in $\theta = [1 - C(A, D^2)]/[N\,C(A, D^2)]$ to obtain the number of rain gages required for each strategy.

If we wish to be able to predict the peak flow, from 2/3 of the way into the flow hydrograph and our estimation error variance is not to exceed 0.5 of the peak flow variance, then Figs. 8 and 9 are appropriate. From Fig. 4 corresponding to $\alpha k = 1.7$ and $m = 3$ we read $\alpha t_m = 7.5$ which results in $t_m = 3.8$ h, i.e., we are selecting strategies to predict a peak that occurs 3.8 h after the start of the storm from 2.5 h into the storm. Strategies for points K through O on Fig. 8 and P through T on Fig. 9 are given in Tables 4 and 5. In these tables the trade-off between rainfall measurement and flow measurement is evident. These

TABLE 3

Example of selection of sampling strategy for total discharge error variance with the Poisson rainfall model

Point	θ	Rainfall sampling frequency $1/(\alpha\Delta t_p)$	Discharge sampling frequency $1/(\alpha\Delta t_q)$	Rainfall sampling interval (h) Δt_p	Discharge sampling interval (h) Δt_q	Number of rain gages N
F	0	0	0.153	∞, none	3.27	0
G	2	0.4	0.145	1.25	3.45	$2.8 \simeq 3$
H	0.1	0.4	0.105	1.25	4.76	57
I	0.5	0.4	0.132	1.25	3.79	11
J	0.1	1	0.094	0.53	5.32	57

TABLE 4

Example of selection of sampling strategy for prediction of peak error variance, with Poisson rainfall model

Point	θ	Rainfall sampling frequency $1/(\alpha \Delta t_p)$	Discharge sampling frequency $1/(\alpha \Delta t_q)$	Rainfall sampling interval (h) Δt_p	Discharge sampling interval (h) Δt_q	Number of rain gages N
K	–	0	0.33	∞, none	1.5	0
L	2	1.0	0.18	0.5	2.8	$2.8 \simeq 3$
M	2	0.4	0.23	1.3	2.2	$2.8 \simeq 3$
N	0.5	0.34	0.0	1.5	∞, none	$11.3 \simeq 11$
O	0.1	0.22	0.0	2.3	∞, none	57

strategies all provide information with the same error variance in each case, so from these the cheapest, or most convenient should be selected. Comparing tables we see that the Poisson model generally requires more sampling. The clustering introduces correlation into the model which allows uncertainty to be reduced to the same level by fewer, or less frequent observations.

CONCLUSIONS

Parameterizations of rainfall and basin response which are simple and allow the use of linear systems theory for analyzing the problem of combined rainfall and runoff measurement have been given. To provide a minimum variance linear estimate of flow from a rainfall event using rainfall and runoff measurements combined, a state space approach has been developed.

The results obtained relate the variance of estimation error to the measurement strategy plus basin and rainfall parameters. Estimation error occurs due

TABLE 5

Example of selection of sampling strategy for prediction of peak error variance with clustered rainfall model

Point	θ	Rainfall sampling frequency $1/(\alpha \Delta t_p)$	Discharge sampling frequency $1/(\alpha \Delta t_q)$	Rainfall sampling interval (h) Δt_p	Discharge sampling interval (h) Δt_q	Number of rain gages N
P	–	0	0.26	∞, none	1.9	0
Q	2	0.4	0.1	1.25	5	$2.8 \simeq 3$
R	2	0.55	0	0.91	∞, none	$2.8 \simeq 3$
S	0.5	0.25	0	2	∞, none	$11.3 \simeq 11$
T	10	0.5	0.23	1	2.2	$0.57 \simeq 1$

to process variability, sampling scarcity and measurement errors. Although the procedure includes measurement error the results presented had measurement error set to zero to maintain generality and reduce the number of parameters being considered. These results could be useful in the design of measurement networks.

This work has limitations in that it assumes linearity in basin response and is dependent on the accuracy of the model and parameters. While the basin response parameters may be obtained from the basin geomorphology, the choice of climate parameter values is not easy. Another deficiency is that the effect of uncertainty in the infiltration, thus runoff, has not been accounted for. We are investigating ways of doing this through incorporation of the contributing area concept into the state space formulation.

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REFERENCES

Austin, P.M. and Houze, R.A., 1972. Analysis of the structure of precipitation patterns in New England. J. Appl. Meteorol., 11: 926–935.

Bras, R.L. and Rodriguez-Iturbe, I., 1975. Rainfall runoff as spatial stochastic processes: data collection and synthesis. Ralph M. Parsons Lab., Tech. Rep. No. 196, Dep. Civ. Eng., MIT, Cambridge, Mass.

Brown, R.G., 1983. Introduction to Random Signal Analysis and Kalman Filtering. Wiley, New York, N.Y., 356 pp.

Fennessey, N., Eagleson, P.S., Wang, Q. and Rodriguez-Iturbe, I., 1986. Spatial analysis of storm depths from an Arizona raingauge network. Ralph M. Parsons Lab., Tech. Rep. No. 306, Dep. Civ. Eng., MIT, Cambridge, Mass.

Harrold, T.W., 1973. Mechanisms influencing the distribution of precipitation within baroclinic disturbances. Q. J. R. Meteorol. Soc., 99: 232–251.

Lenton, R.L. and Rodriguez-Iturbe, I., 1974. On the collection, the analysis, and the synthesis of spatial rainfall data. Ralph M. Parsons Lab., Tech. Rep. No. 194, Dep. Civ. Eng., MIT, Cambridge, Mass.

Nash, J.E., 1957. The form of the instantaneous unit hydrograph. IAHS–AISH Publ., (42): 114–118.

Rodriguez-Iturbe, I. and Eagleson, P.S., 1987. Mathematical models of rainstorm events in space and time. Water Resour. Res., 23(1): 181–190.

Rodriguez-Iturbe, I. and Mejia, J.M., 1974. The design of rainfall networks in time and space. Water Resour. Res., 10(4): 713–728.

Rodriguez-Iturbe, I. and Valdes, J.B., 1979. The geomorphologic structure of hydrologic response. Water Resour. Res., 15(5): 1409–1420.

Rosso, R., 1984. Nash model relation to Horton order ratios. Water Resour. Res., 20(7): 414–920.

Strahler, A.N., 1964. Quantitative geomorphology of drainage basins and channel networks. In: V.T. Chow (Editor), Handbook of Applied Hydrology. McGraw-Hill, New York, N.Y., Sect. 4-II.

Tarboton, D.G., 1987. Hydrological sampling: A characterization in terms of rainfall and basin properties. M.S. Thesis, submitted to the Dep. Civ. Eng., MIT, Cambridge, Mass.

Tarboton, D.G., Bras, R.L. and Puente, C.E., 1987. Combined hydrologic sampling criteria for rainfall and streamflow. J. Hydrol., 95: 323–339.

Valdes, J.B., Rodriguez-Iturbe, I. and Gupta, V.K., 1985. Approximations of temporal rainfall from a multidimensional model. Water Resour. Res., 21(8): 1259–1270.

Waymire, E., Gupta, V.K. and Rodriguez-Iturbe, I., 1984. A spectral theory of rainfall intensity at the Meso-β scale. Water Resour. Res., 20(10): 1453–1465.